Hand-In Assignment 5

1. Decide whether the series converges or diverges in $\mathbb{R}$.

   a) $\sum_{n=1}^{\infty} \frac{n}{n^3+1}$ [2 pts]

   b) $\sum_{n=1}^{\infty} \frac{n^{2n}}{\left(1+2n^2\right)^n}$ [2 pts]

   c) $\sum_{n=1}^{\infty} \frac{Cos(3n)}{1+(1.2)^n}$ [2 pts]

   d) $\sum_{n=1}^{\infty} \frac{1\cdot 3\cdot 5\cdot \ldots \cdot (2n-1)}{5^n\, n!}$ [2 pts]

   e) $\sum_{n=1}^{\infty} \ln\left(\frac{n}{3n+1}\right)$ [2 pts]

2. Find the exact sum of the series $\sum_{n=1}^{\infty}\left(Sin\left(\frac{\pi}{2}-\frac{\pi}{n+1}\right)-Sin\left(\frac{\pi}{2}-\frac{\pi}{n}\right)\right)$ [10 pts]

3. Let $\{a_n\}_{n=1}^{\infty}$ be a sequence defined by $a_n = \begin{cases} \frac{1}{2}+\frac{1}{2}\left(1-\frac{1}{k}\right) & if \quad n=2k-1 \\ 2-\frac{1}{2}\left(1-\frac{1}{k}\right) & if \quad n=2k \end{cases}$,

   where $k \geq 1$. Calculate $\limsup\{a_n\}$ and $\liminf\{a_n\}$. [10 pts]

4. Compute $\limsup\{a_n\}$ and $\liminf\{a_n\}$ for the sequence

   $a_n = Sin\left(\frac{\pi}{2}n\right)\frac{n+2}{2n}$. [10 pts]