Hand-In Assignment 4

1. Let G be an open subset of (M, d) and F be a closed subset of (M, d). Prove or disprove: G\F is an open subset of (M, d).

2. Let $G = \{(x, y) \in \mathbb{R}^2 : x \neq y\}$. Prove that G is an open subset of $\mathbb{R}^2$.
[10 pts]

3. Suppose that E is a nowhere dense subset of (M, d). Prove or disprove: E^c is everywhere dense in M. [10 pts]

4. Let ◊ be a set constructed out of the interval [0, 1] as follows:

   Step 1: Partition the interval into 5 parts of equal length and remove every other (open) part. Thus, you obtain the set
   $I_1 = [0, 1/5] \cup [2/5, 3/5] \cup [4/5, 1]$

   Step 2: Partition each of the interval segments of I_1 further into 5 parts of equal length and remove every other (open) part. Thus, you obtain the set
   $I_2 = [0, 1/25] \cup [2/25, 3/25] \cup [4/25, 1/5] \cup$
   $\cup [2/5, 11/25] \cup [12/25, 13/25] \cup [14/25, 3/5] \cup$
   $\cup [4/5, 21/25] \cup [22/25, 23/25] \cup [24/25, 1]$

   Step n: Partition each of the interval segments of I_{n-1} into 5 parts of equal length and remove every other (open) part to obtain I_n.

   Set $\Diamond = \bigcap_{n=1}^{\infty} I_n$.

   a) Is ◊ closed as a subset of $\mathbb{R}$? [2 pts]
   b) Is ◊ countable or uncountable? [2 pts]
   c) Is ◊ dense, nowhere dense, or neither? [2 pts]
   d) Is ◊ perfect? [2 pts]
   e) What is the "length" (i.e measure) of ◊? [2 pts]
   f) The elements of ◊ can be most easily described in terms of some base p decimal expansion. What p should we choose? In terms of the decimal expansion base p, how would you decide whether x is an element of ◊?
   [10 pts]
   g) Construct a Cantor-like function for ◊. [5 pts]