Hand-In Assignment 3

1. Which of the following are metric functions on $(0, \infty)$? Write simply "metric" or "not metric".

   a) $d(x, y) = \left| \frac{1}{x^4} - \frac{1}{y^4} \right|$ [1 pt]

   b) $d(x, y) = |x - 3y|$ [1pt]

   c) $d(x, y) = \sqrt{|x - y|} + \frac{|x - y|}{1 + |x - y|}$ [1 pt]

   d) $d(x, y) = \tan^{-1}|x - y|$ [1 pt]

   e) $d(x, y) = \min\left\{ |x - y|^{3/4}, 2 \right\}$ [1 pt]

   Which of the following are metric functions on $(0, \infty) \times (0, \infty)$? Write simply "metric" or "not metric".

   f) $d((x, y), (w, z)) = \sqrt{\left| \frac{1}{x^4} - \frac{1}{w^4} \right|^2 + \left| \frac{1}{y^4} - \frac{1}{z^4} \right|^2}$ [1 pt]

   g) $d((x, y), (w, z)) = |x - 3w| + |y - z|$ [1 pt]

   h) $d((x, y), (w, z)) = \sqrt{|x - w|} + \frac{|y - z|}{1 + |y - z|}$ [1 pt]

   i) $d((x, y), (w, z)) = \tan^{-1}\left(\sqrt{|x - w|^2 + |y - z|^2} \right)$ [1 pt]

   j) $d((x, y), (w, z)) = \min\left\{ |x - w|^{3/4}, 2 \right\} + \min\left\{ |y - z|^{1/4}, 1 \right\}$ [1 pt]

2. Let M = (0, ∞) be supplied with the metric function $d(x,\ y) = \left|\frac{1}{x} - \frac{1}{y}\right|$ and let $\{n\}_{n=1}^{\infty}$ be a sequence of positive integers.
   a) Is the sequence $\{n\}_{n=1}^{\infty}$ a Cauchy sequence in (M, d)? Justify your answer. [6 pts]
   b) Does the sequence $\{n\}_{n=1}^{\infty}$ converge in (M, d)? Justify your answer. [4 pts]

3. True or false? $\left|\tan^{-1}|x| - \tan^{-1}|y|\right| \le \tan^{-1}|x - y|$ Justify your answer. [Hint: look at HW # 3, problem 1] [10 pts]

4. Let (ℝ, d) be a metric space with the metric function $d(x,\ y) = \frac{|x - y|}{1 + |x - y|}$. Calculate diam(0, ∞). [10 pts]