Hand-In Assignment 3 (Advanced)

1. Let C[a, b] be the space of continuous functions. For $f \in C[a,\ b]$ and $p \geq 1$, define $\|\cdot\|_p$ by $\|f\|_p = \left(\int_a^b |f(t)|^p \, dt \right)^{1/p}$. State and prove Hoelder's Inequality for $\|\cdot\|_p$.

[10 pts]

2. State and prove Minkowki's Inequality for $\|\cdot\|_p$ as it is defined above.

[10 pts]

3. Use Hoelder's Inequality to show that for all $f \in C[0,\ 1]$ and all 1 ≤ r < s, $\|f\|_r \leq \|f\|_s$. [Hint: Notice that $\|f\|_r^r = \langle 1,\ |f|^r \rangle = \int_0^1 1 \cdot |f(t)|^r \, dt$. By Hoelder's Inequality, $|\langle 1,\ |f|^r \rangle| \leq \|1\|_q \|f\|_p$. Now what are suitable choices for the pair q, p?]

[20 pts]

Remark: The norm $\|\cdot\|_p$ "works" on some non-continuous functions as well. The space of all p-integrable real-valued functions on R is called L_p space.