Hand-In Assignment 1

Let $A, B \subset \mathbb{R}$ be nonempty.

1. Define $A + B = \{x + y : x \in A \text{ and } y \in B\}$.
   Compute sup $(A + B)$ in terms of sup (A) and sup (B). Repeat exercise for inf $(A + B)$. Justify your answer. [10 pts]

2. Let $c > 0$. Define $cA = \{c\, x: x \in A\}$. Compute sup (cA) in terms of sup (A). What happens if $c < 0$? Repeat exercise for inf (cA). [10 pts]

3. Define $AB = \{xy: x \in A \text{ and } y \in B\}$. Assuming that the elements of A and the elements of B are nonnegative, compute sup (AB) in terms of sup (A) and sup (B). Is your answer still true if we drop the assumption that A and B are nonnegative? [10 pts]

4. Suppose $f : A \to \mathbb{R}$ and $g : A \to \mathbb{R}$ are real valued functions. Define $f(A) \oplus g(A) = \{f(x) + g(x): x \in A\}$ and $f(A) + g(A) = \{f(x) + g(y): x, y \in A\}$.
   What is the relationship between sup $(f(A) \oplus g(A))$ and sup $(f(A) + g(A))$?
   Repeat exercise for inf $(f(A) \oplus g(A))$ [10 pts]