Review for Exam 2

1) Determine whether the function is injective, surjective, and / or bijective.

(a) $f: [0, \infty) \to \mathbb{R};\ f(x) = x + 5$

(b) $g: \mathbb{R} \to \mathbb{R};\ g(x) = x + 5$

(c) $h: [0, \infty) \to [0, \infty);\ h(x) = 25x^2$

(d) $k: [0, \infty) \to \mathbb{R};\ k(x) = 25x^2$

(e) $l: \mathbb{R} \to [0, \infty);\ k(x) = 25x^2$

(f) $p: \mathbb{R} \to \mathbb{R};\ g(x) = 3x - 9$

(g) $q: (0, \infty) \to (0, \infty);\ g(x) = \frac{1}{x}$

2) Find the inverse

(a) $f: \mathbb{R} \to \mathbb{R};\ f(x) = x + 3$

(b) $g: \mathbb{R} \to \mathbb{R};\ f(x) = 2x - 5$

(c) $h: (0, \infty) \to (0, \infty);\ h(x) = \frac{1}{3x}$

(d) $k: (0, \infty) \to \left(\frac{1}{5}, \infty\right);\ h(x) = \frac{1}{2x+5}$

(e) $p: \mathbb{R} - \left\{\frac{3}{2}\right\} \to \mathbb{R} - \{2\}; p(x) = \frac{4x+7}{2x-3}$

3) Show that the following sets are infinitely countable. That is, the Hilbert Hotel can accommodate all the members of the set.

(a) $A \cup B$, where $A = \{a_1, a_2, a_3, \dots\}$ and $B = \{b_1, b_2\}$

(b) $A \cup B$, where $A = \{a_1, a_2, a_3, \dots\}$ and $B = \{b_1, b_2, b_3, \dots\}$

(c) $A_1 \cup A_2 \cup A_3 \cup \dots$ where each A_k is a countable infinite set. i.e. Is the countable union of countable sets countable?

(d) $A \times B = \{(a, b): a \in A, b \in B\}$ where both A and B are infinitely countable.

4) Use Cantor's diagonalization argument to produce a sequence nowhere on the list

(1) 0.11223344011...

(2) 0.21212121212...

(3) 0.31323255555...

(4) 0.41429333946...

(5) 0.52323445185...

(6) 0.61277459881...

5) Explain why the set of real numbers is not countable. Specifically, explain why the set of real numbers is the interval (0, 1) is uncountable.

6) Use the language of functions to explain what it means for the cardinality of a set A to be less that the cardinality of a set B.

7) Show that every infinite set contains an infinitely countable subset.

8) Let $A = \{a, b, c, d\}$ and $F: A \to \mathcal{P}(A)$. In each of the following cases, compute $S = \{x \in A: x \notin F(x)\}$

(a) $F(x) = \begin{cases} \{a, b\} & x = a \\ \{a\} & x = b \\ \{a, d\} & x = c \\ \{b, c\} & x = d \end{cases}$

(b) $F(x) = \begin{cases} \{b, d\} & x = a \\ \emptyset & x = b \\ \{a, d\} & x = c \\ \{a, b, c, d\} & x = d \end{cases}$

(c) $F(x) = \begin{cases} \{a\} & x = a \\ \{b\} & x = b \\ \{c\} & x = c \\ \{d\} & x = d \end{cases}$

9) Show that for any set A, the power set $\mathcal{P}(A)$ always has bigger cardinality.

10) Explain why countable infinity is the smallest infinity there is. Does there exist a biggest infinity? Why or why not?