# Independent Events

Generally, when an event $F$ occurs, $F$ modifies the likelyhood of another event $E$. That is
$P(E|F) \neq P(E)$

If it happens that $P(E|F) = P(E)$ we say that $E$ is independent of $F$.

Notice that $P(E|F) = \dfrac{P(EF)}{P(F)} = P(E)$ whenever $P(F) \neq 0$. Hence $\boxed{P(EF) = P(E)P(F)}$

To avoid the fuss of considering whether or not $P(F)=0$ Independence is defined to be the property $P(EF) = P(E)P(F)$. When it holds, we say that $E$ and $F$ are independent.

<u>Observation:</u> If $P(E) \neq 0$ and $P(F) \neq 0$ and $P(E|F) = P(E)$ then $P(F|E) = P(F)$

<u>proof:</u> $P(F|E) = \dfrac{P(EF)}{P(E)} = \dfrac{P(E)P(F)}{P(E)} = P(F)$

<u>Observation:</u> If $E$ and $F$ are independent then so are $E$ and $F^c$

Proof: $P(EF^c) = p(E) - p(EF) = p(E) - p(E)p(F)$
$= p(E)(1 - p(F)) = p(E)p(F^c)$.

Ex. 2 Fair dice tossed. $E_1$ - sum $= 6$, $E_2$ - sum $= 7$

$F$ - First die $= 4$.

Q. Is $E_1$ independent from $F$?

A. $P(E_1) = \frac{5}{36}$ $\quad P(E_1|F) = \frac{1}{6}$

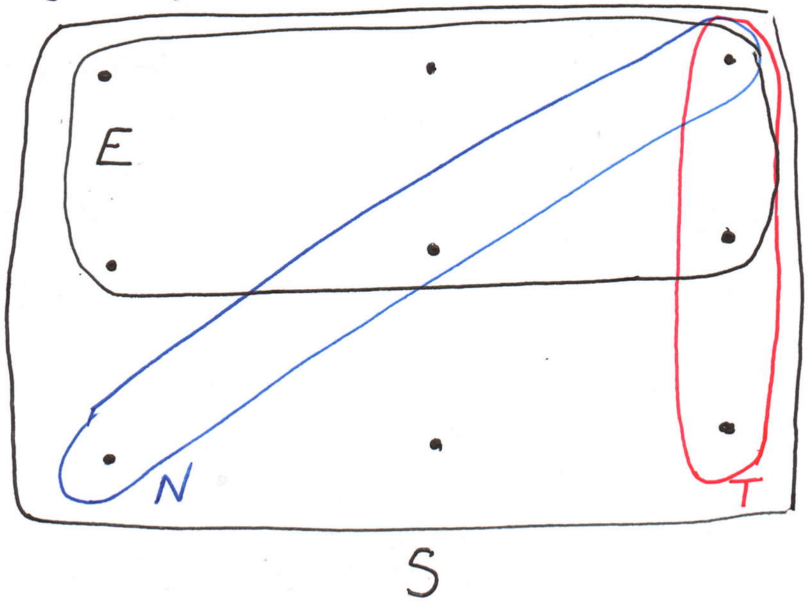
$p(E_1) < \frac{6}{36} = \frac{1}{6} = p(E_1|F)$. No.

Q. Is $E_2$ independent from $F$?

A. $P(E_2) = \frac{6}{36} = \frac{1}{6}$ $\quad P(E_2|F) = \frac{1}{6}$ Yes

Q. Suppose $E$ is independent from $N$ and that $E$ is independent from $T$. Is $E$ independent from $NT$?

A. Little Red Ridinghood meats several animals on her way to grandma from the forrest. (9 animals to be exact). 6 of the animals have big eyes, Three of them have long noses, and three of them have sharp teeth. Having taken my probability class, she draws the following diagram.




She then wonders

(a) what is the probability he has big eyes given that he has a long nose to sniff me?

$P(E|N) = \frac{2}{3}$ ; $P(E) = \frac{6}{9} = \frac{2}{3}$.

Phew! E and N are independent. It didn't get more dangerous just yet.

(b) Now what's the probability he has big eyes given that his teeth are so sharp?

$P(E|T) = \frac{2}{3}$ ; $P(E) = \frac{2}{3}$.

Phew! E and T are independent. It didn't get more dangerous just yet.

(c) Now what's the probability he has big eyes given that he has a sharp nose and teeth?

$p(E|NT) = 1$  !!!

As my brother likes to say. He is looking at Eu-clid.

<u>Def:</u> Three sets are said to be independent if

$$p(EFG) = p(E)p(F)p(G)$$
$$p(EF) = p(E)p(F)$$
$$p(EG) = p(E)p(G)$$
$$p(FG) = p(F)p(G).$$