NAME:

Fall 2019 Stat 311 Exam 3

Instructions: WRITE YOUR NAME CLEARLY. Do as many problems as you can for a maximal score of 100. SHOW YOUR WORK!

1. The amount of time that a computer functions before breaking down is a continuous random variable with probability density function given by

$$f(x) = \begin{cases} \lambda e^{-x/100} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

What is the probability that a computer will function between 50 and 150 hours before breaking down? [10 pts]

2. Find $E[X^2]$ where the probability density function of X is given in problem 1. [10 pts]

3. The density function of X is given by $f(x) = \begin{cases} 1 & 0 \le x \le 1 \\ 0 & otherwise \end{cases}$ Find the density function of $Y = e^X$ [10 pts]

4. An accident is equally likely to occur anywhere on a strip of highway $(0, L)$. At the moment of the accident, the dispatched ambulance is equally likely to be located anywhere in the strip $(0, L)$ and its position is independent from the place of accident. Find the probability density function of the distance of the ambulance from the accident. [10 pts]

5. Suppose that the height, in inches, of a 25-year-old man is a normal random variable with parameters $\mu = 71$ and $\sigma^2 = 6.25$. What percentage of 25-year-old men are over 6 feet, 2 inches tall? [Hint: 1 foot = 12 inches]   [10 pts]

6. Each item produced by a certain manufacturer is, independently, of acceptable quality with probability 0.95. Approximate the probability that at most 10 of the next 150 items produced are unacceptable.   [10 pts]

7. Three points X, Y, Z are selected at random on a line L. What is the probability that Y lies between X and Z? [10 pts]

8. The joint density function of X and Y is given by

$$f(x,y) = \begin{cases} 24xy & 0 \le x \le 1, 0 \le y \le 1 - x \\ 0 & otherwise \end{cases}$$

Compute $P(Y > 1/2 | X < 1/2)$ [10 pts]

9. The density function of X is given by $f(x) = \begin{cases} a + bx^2 & 0 \le x \le 1 \\ 0 & otherwise \end{cases}$

   If E[X] = 3/5, find a and b. [10 pts]

10. The monthly worldwide average number of airplane crushes is 2.2. What is the probability that there will be more than 3 such accidents in the next 2 months? [10 pts]

Extra-Credit

11. If Y is uniformly distributed over (0, 5), what is the probability that the roots of the equation $4x^2 + 4xY + Y + 2 = 0$ are both real? [10 pts]

12. For reasons that are beyond my waking comprehension my NJ cousin prefers Dominos over oven baked pizza located nearby. He also likes it with pepperoni , but orders the pizza half plane as a compromise with his 'favorite' guest. If the pizza baker scatters n pepperoni slices at random before cutting the pie, what's the probability that the pizza can be divided in half through its diameter, with all the pepperoni slices on one of the halves? You may assume that the pepperoni slices are points. Calling any item on this pizza "plane" would be a huge overstatement! [10 pts]

Tables of the Normal Distribution

Probability Content from   -oo to Z

Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990