Name:

Spring 2020 Stat 311 Exam 2

Instructions: WRITE YOUR NAME CLEARLY. Do as many problems as you can for a maximal score of 100. SHOW YOUR WORK!

1. A boy carries 3 white and 2 red marbles in his left pocket and 4 white and 1 red marbles in his right pocket. The boy reaches randomly into one pocket and draws out one marble. Given that a randomly picked marble is white, what is the probability it came out of the left pocket? [10 pts]

2. It has been observed that an average of 3 crabs in a crab swarm are eaten every minute by their predators. Estimate the probability that 5 crabs will be eaten in the next 2 minutes [10 pts]

3. In a country of 300 million people, 3 million are assumed to have contracted a scary illness. A test to determine whether or not a given patient has the disease is 99% reliable. If one wishes to know whether or not he or she is infected, what is the better strategy? (a) Go get tested? (b) Flip a coin? Explain. [10 pts]

4. 3 cards are randomly chosen without replacement from an ordinary deck of 52 cards. Given that all the chosen cards are aces, what's the probability that an ace of spades is among these 3 cards? [10 pts]

5. 100 students are randomly divided into pairs on day 1. They are randomly divided into pairs on day 2 again. Let X = 0, 2, 4,..., 100 be the number of students that had the same partner on both days. Compute $E[X]$. [10 pts]

6. A blogger discovers two peculiar clubs. The Randomly Social Regressive and the Randomly Social Progressive clubs appear to be very different indeed!

(a) The first club thinks it's the 1950s and goes by the motto "A man is a man and a woman is a woman". The blogger observes that there are 10 people waiting to enter the club, 6 of which are assigned the gender "woman" and 4 of which are assigned the gender "man". Thereupon 5 people are randomly admitted in . If X is the number of women that went in, compute P(X = k). [4 pts]

(b) The second club feels that people are randomly free to determine their gender once they cross the threshold to the club. The blogger observes that 10 people were waiting to enter. The first 5 are let in, thereupon 6 of the 10 randomly assume the gender "woman" and 4 randomly assume the gender "man". If Z is the number of women in this club, compute $P(Z = k)$. [4 pts]

(c) Are the random variables X and Z really any different? Explain. [2 pts]

7. You visit a family known to have 2 children. You see one child playing in the yard and note that she is a girl. Her mother tells you that she was born on a Wednesday. What is the probability that both children are girls? [10 pts]

8. An ice cream store is selling, on average, 50 boxes of strawberry ice cream per day with standard deviation 5 during the summer season. Use Chebychev's inequality to estimate the probability that on June 19 over 65 or under 35 strawberry ice cream boxes will be sold.

[10 pts]

9. n words are randomly chosen from the set {mortal, coil, this, shuffle, off, to, be, not, or}. How many times do you expect the sentence "to be or not to be" to occur? [10 pts]

10. A and B alternate rolling a pair of dice, stopping either when A rolls the sum 7 or when B rolls the sum 9. Assuming that A rolls first, find the probability that the final roll is made by A.

[10 pts]

Extra Credit

11. Lilli pond leafs labeled 0-10 form a bridge from one shore of the pond to the next. A frog is initially on leaf # 1 and a snake is waiting in ambush on leaf # 0. If the frog jumps to leaf # 0, it is dead, whereas, if it gets to leaf # 10, the frog survives. Given that the frog is on leaf # k, it will jump to leaf # k-1 with probability $\frac{k}{10}$ and it will jump to leaf # k+1 with probability $\frac{10-k}{10}$. For example, given that the frog is on leaf # 3, it will jump back to leaf # 2 with probability $\frac{3}{10}$ and it will jump forward to leaf number 4 with probability $\frac{10-3}{10} = \frac{7}{10}$. Find the probability that the frog survives.

										
0	1	2	3	4	5	6	7	8	9	10

[10 pts]

12. There are 100 equally spaced points around the circle. At 99 points, there are sheep, and at one point, there is a wolf. At each time step, the wolf randomly moves either clockwise or counterclockwise by 1 point. If there is a sheep at that point, the wolf eats it. The sheep don't move. What is the probability that the sheep who is initially opposite the wolf is the last one remaining? [10 pts]