

**Name:**

## Fall 2019 Stat 311 Exam 2

**Instructions:** WRITE YOUR NAME CLEARLY. Do as many problems as you can for a maximal score of 100. SHOW YOUR WORK!

1. A boy carries 3 white and 2 red marbles in his left pocket and 4 white and 1 red marbles in his right pocket. The boy reaches randomly into one pocket and draws out one marble. Given that a randomly picked marble is white, what is the probability it came out of the left pocket?

[10 pts]

2. An urn contains 4 white and 8 red balls. 3 balls are taken out at random. What is the expected number of white balls in this sample?

[10 pts]

3. In a certain college, 80% of instructors take a shot of vodka after grading exams. Of those instructors that take a shot of vodka, 50% also take a shot of whisky. What percentage of instructors take a shot of vodka and a shot of whisky? [10 pts]

4. 3 cards are randomly chosen without replacement from an ordinary deck of 52 cards. Given that all the chosen cards are aces, what's the probability that an ace of spades is among these 3 cards? [10 pts]

5. Let  $X$  be a discrete random variable that takes on values  $\frac{1}{n}$ ,  $n = 1, 2, 3, \dots$  with  $P\left(X = \frac{1}{n}\right) = \frac{2}{3^n}$ . Compute  $E\left[\frac{1}{X}\right]$ . [10 pts]

6. Let  $X$  be a nonnegative random variable with cumulative distribution function  $F$  given by

$$F(x) = \begin{cases} \frac{3}{25}x & \text{if } 0 \leq x < 5 \\ \frac{4}{5} & \text{if } 5 \leq x < 6 \\ 1 & \text{if } 6 \leq x \end{cases}$$

What is  $P(X = 5)$ ?

[10 pts]

7. A ball is equally likely to be painted yellow or blue. Two such balls are placed in an urn. Given that one of the balls is yellow, what's the probability that both balls are yellow? [10 pts]

8. An ice cream store is selling, on average, 50 boxes of strawberry ice cream per day with standard deviation 5 during the summer season. Use Chebychev's inequality to estimate the probability that on June 19 over 65 or under 35 strawberry ice cream boxes will be sold.



[10 pts]

9.  $n$  words are randomly chosen from the set {mortal, coil, this, shuffle, off, to, be, not, or}. How many times do you expect the sentence "to be or not to be" to occur? [10 pts]

10. A and B alternate rolling a pair of dice, stopping either when A rolls the sum 7 or when B rolls the sum 9. Assuming that A rolls first, find the probability that the final roll is made by A. [10 pts]

### Extra Credit

11. Lilli pond leaves labeled 0-10 form a bridge from one shore of the pond to the next. A frog is initially on leaf # 1 and a snake is waiting in ambush on leaf # 0. If the frog jumps to leaf # 0, it is dead, whereas, if it gets to leaf # 10, the frog survives. Given that the frog is on leaf #  $k$ , it will jump to leaf #  $k-1$  with probability  $\frac{k}{10}$  and it will jump to leaf #  $k+1$  with probability  $\frac{10-k}{10}$ . For example, given that the frog is on leaf # 3, it will jump back to leaf # 2 with probability  $\frac{3}{10}$  and it will jump forward to leaf number 4 with probability  $\frac{10-3}{10} = \frac{7}{10}$ . Find the probability that the frog survives.

										
0	1	2	3	4	5	6	7	8	9	10

[10 pts]

12. There are 100 equally spaced points around the circle. At 99 points, there are sheep, and at one point, there is a wolf. At each time step, the wolf randomly moves either clockwise or counterclockwise by 1 point. If there is a sheep at that point, the wolf eats it. The sheep don't move. What is the probability that the sheep who is initially opposite the wolf is the last one remaining? [10 pts]