Name:

Fall 2020 Stat 311 Exam 1

Instructions: WRITE YOUR NAME CLEARLY. Do as many problems as you can for a maximal score of 100. SHOW YOUR WORK! You are not presenting a valid Kafka Protocol at your peril.

 One of my cats, Shpuntik (Шпунтик), is a fussy eater. Give him the same dish more than once in an interval of 3 days and he won't eat it! If I have an assortment of 5 dish types (10 cans each), how many menus can I make for the next 10 days? Assume that Shpuntik is getting one can a day. [10 pts]

2. 200 passengers have booked a flight from NY to London. Each passenger was naturally assigned a seat number, but the first passenger to board has decided to randomly pick a seat. The remaining passengers are British and therefore too polite: Each subsequent passenger to board will take up his or her assigned seat if it is available and randomly pick a seat among the still empty places if not. What is the probability that the last passenger to board will be in his assigned seat? [10 pts]

3. A deck of 52 cards is randomly shuffled and the cards are turned over one by one until the first ace appears. What is the probability that no King, Queen, or Jack appear before the first ace?
[10 pts]

4. An inspector checks 12 identically looking transistors for defects. He finds that 3 transistors are malfunctioning and sets them aside for disposal. Unfortunately his apprentice mixes them back with the rest by mistake. What is the probability the inspector will have to test 9 or more transistors before locating the 3 broken ones among the 12? [10 pts]

32 cards are randomly drawn from an ordinary deck of 52 cards and divided equally among 4 players. What is the probability that exactly 2 players receive all 8 cards of the same suit? E.g. player 1 receives 8 hearts and player 2 gets 8 spades. [10 pts]

6. Julius Caesar is rumored to have said "Et tu, Brute?" with his last dying breath. If each breath contains about 10²² molecules and these molecules of last breath are thoroughly mixed among the 10⁴⁴ molecules in the atmosphere, estimate the probability that you are inhaling at least one of these historical molecules at this moment. You may assume for simplicity that the molecules in your breath are inhaled (sampled) sequentially with replacement. [10 pts]

7. A fair coin is flipped until either HH or TH appears for the first time. Is one pattern more likely than another? Compute the probabilities. [10 pts]

k balls are randomly and sequentially drawn from a bowl containing n balls numbered ball #1 - ball #n . If the drawing is done with replacement, compute the probability of obtaining a monotone, non-decreasing sequence. E. g. 1, 2, 2, 2, 4, 5, 5, ... [10 pts]

9. Two decks of 52 cards are randomly shuffled together to form a deck of 104 cards. What is the probability that no two copies of the same card are next to each other? [10 pts]

10. 5 chess players will each play one match against the other. How many outcomes are possible if each outcome is to record the result of every match? Each match can result in a win for one player or else it is a draw. [10 pts]

Extra Credit

11. How many permutations of {1, 2, 3, ..., n} are possible if each number cannot be moved farther than a distance k from each original place, where k is a nonnegative integer? [10 pts]

12.Simplify the product $(2n-1) \cdot (2n-3) \cdot (2n-5) \cdots 3 \cdot 1$ by realizing what is being counted
and using a different counting procedure. In short, come up with a combinatorial argument.
(Hint: How many ways are there for 2n students to break into groups of two?)[10 pts]