



3. A deck of 52 cards is randomly shuffled and the cards are turned over one by one until the first ace appears. What is the probability that no King, Queen, or Jack appear before the first ace? [10 pts]
4. An inspector checks 12 identically looking transistors for defects. He finds that 3 transistors are malfunctioning and sets them aside for disposal. Unfortunately his apprentice mixes them back with the rest by mistake. What is the probability the inspector will have to test 9 or more transistors before locating the 3 broken ones among the 12? [10 pts]

5. 32 cards are randomly drawn from an ordinary deck of 52 cards and divided equally among 4 players. What is the probability that exactly 2 players receive all 8 cards of the same suit? E.g. player 1 receives 8 hearts and player 2 gets 8 spades. [10 pts]
6. Julius Caesar is rumored to have said "Et tu, Brute?" with his last dying breath. If each breath contains about  $10^{22}$  molecules and these molecules of last breath are thoroughly mixed among the  $10^{44}$  molecules in the atmosphere, estimate the probability that you are inhaling at least one of these historical molecules at this moment. You may assume for simplicity that the molecules in your breath are inhaled (sampled) sequentially with replacement. [10 pts]

7. A fair coin is flipped until either HH or TH appears for the first time. Is one pattern more likely than another? Compute the probabilities. [10 pts]
8.  $k$  balls are randomly and sequentially drawn from a bowl containing  $n$  balls numbered ball #1 - ball # $n$ . If the drawing is done with replacement, compute the probability of obtaining a monotone, non-decreasing sequence. E. g. 1, 2, 2, 2, 4, 5, 5, ... [10 pts]

9. Two decks of 52 cards are randomly shuffled together to form a deck of 104 cards. What is the probability that no two copies of the same card are next to each other? [10 pts]

10. 5 chess players will each play one match against the other. How many outcomes are possible if each outcome is to record the result of every match? Each match can result in a win for one player or else it is a draw. [10 pts]

**Extra Credit**

11. How many permutations of  $\{1, 2, 3, \dots, n\}$  are possible if each number cannot be moved farther than a distance  $k$  from each original place, where  $k$  is a nonnegative integer? [10 pts]
12. Simplify the product  $(2n - 1) \cdot (2n - 3) \cdot (2n - 5) \cdots 3 \cdot 1$  by realizing what is being counted and using a different counting procedure. In short, come up with a combinatorial argument. (Hint: How many ways are there for  $2n$  students to break into groups of two?) [10 pts]