Name:

Fall 2021 Stat 311 Final

Instructions: Submit as a single PDF file titled P Last Name First Name Final (e.g. P Etkin Arkady Final). Don't forget the P in front or you will turn into a goat! Do as many problems as you can for a maximal score of 100. SHOW YOUR WORK! You are not presenting a valid Kafka Protocol at your peril.

1.   A researcher observes one very curious phenomenon; when he displays the phrase

dont cheat on exam last semester i caught eight

before test subjects, the message appears to them as randomly scrambled noise (e.g. subject may see **ondtacethnomaxestlasemtsereciahtguiethg**). If the letters are randomly permuted, how many letters on average will end up in their proper place?

[Hint: Be careful!]   [10 pts]

2.   For reasons that are beyond my waking comprehension my NJ cousin prefers Dominos over oven baked pizza located nearby. He also likes it with pepperoni , but orders the pizza half plane as a compromise with his 'favorite' guest. If the pizza baker scatters n pepperoni slices at random before cutting the pie, what's the probability that the pizza can be divided in half through its diameter, with all the pepperoni slices on one of the halves? You may assume that the pepperoni slices are points. Calling any item on this pizza "plane" would be a huge overstatement!   [10 pts]

3.   Every given minute an ameba is equally likely to do one of 3 things: (1) Die (2) Divide into two (3) Stay the same. If the ameba divides, each of the two cells will continue the above cycle independently. What is the probability that the ameba colony generated from a single cell will eventually die out?   [10 pts]

4.   A catering company is planning to supply a banquet of 300 people. Each guest will have the choice between dish A and dish B. To estimate how many portions of dish A the company should make, the company invites 20 people to sample the two dishes.

(a) If 15 out of the 20 people prefer dish A to dish B, estimate the probability that a random person will order dish A during the banquet. [Hint: Use Beta distribution]   [10 pts]

(b) What is the minimal number of dish A portions the company should prepare in order to insure with 99.9% certainty that the demand does not exceed the supply? [Hint: Central Limit Theorem]   [10 pts]

5.   A deck of 52 cards is divided equally among 4 players. What is the probability that each player will receive all 13 cards of the same suit?   [10 pts]

6.   A stick of length L has a red point q marked on it. If the stick is broken at a point X that is uniformly distributed over $(0, L)$, calculate the expected length of segment with the red point q.   [10 pts]

7. In the Sanatorium For Acute Intellectual Malnutrition, a nourishing twelve course Christmas Exam is finally administered to 20 starving statistics orphans who are seated in a circle. Having been deprived to the point of not feeling hungry, each student randomly shoves the exam to the fellow on his right or the one on his left. How many orphans on average will end up without a test? [10 pts]

8. An urn contains 6 white and 14 red balls. 5 balls are taken out at random. What is the expected number of white balls in this sample? [10 pts]

9. Let X be a continuous random variable with probability density function given by

$$f(x) = \begin{cases} \dfrac{1}{2}x & 0 < x < 2 \\ 0 & otherwise \end{cases}$$

Find the probability density function of $Y = 3X + 1$ [10 pts]

10. Santa Claus Office of Entitlement reports that spoiled children are three times as likely to demand four presents as they are to demand two. If the number of presents a spoiled child desires has a Poisson distribution, what is its variance?   [10 pts]

Extra Credit

The following two questions illustrate why trying to understand real world events through second-hand statistics is like trying to enter Kafka's castle. It is certainly possible to wonder astray following numbers that were pulled out from that abysmal hole the Germans call "der After". It is, however, far more interesting what bureaucrats can make with correct information in their zeal to do us all good on paper. One can be so sweepingly formal as to observe that 98% of all living organisms live under water and drown us all for good health.

11.   The emergency approval of the Pfeizer vaccine has been argued for with the following study: 20,172 participants were assigned to the placebo group and 19,965 participants were injected with the drug. All participants were followed for two months only. Within this time period, 169 participants became symptomatic with Covid in the placebo group as opposed to 9 participants in the vaccine group. Based on this data, Pfeizer declared the efficiency of their drug at 95%.

   (a) Here is my interpretation of this number: Imagine that if all the symptomatic placebo patients were injected with Pfeizer, then all but 9 of them would be cured. Use this to estimate the success rate of the drug at preventing symptoms (for two months).   [5 pts]

   (b) If Pfezer statistics are true, what was the likelihood for a random unvaccinated person to develop symptomatic Covid within the two months? What was the corresponding likelihood in the vaccinated group?   [5 pts]

12. As it happens, the risk for developing severe Covid is vastly different for various age groups as you can see in the next table's Infection Fatality Rates (IFR).

Age	Median IFR	Survival Rate Estimate
0-19	0.0027%	99.9973%
20-29	0.014%	99.9860%
30-39	0.031%	99.9690%
40-49	0.082%	99.9180%
50-59	0.27%	99.7300%
60-69	0.59%	99.4100%

https://thenewamerican.com/stanford-epidemiologist-studies-covid-finds-its-highly-treatable-and-rarely-a-deadly-disease/

(a) Proponents of collective vaccine compliance (including for children above 5) argue that vaccinated individuals are less infectious. Does the following statistical data from Israel support this claim? Explain. [6 pts]

ISRAEL CONFIRMED CASES, JULY 4 TO JULY 31

Age Group	Cases Fully 💉	Cases 🚫	Percent of Cases Fully 💉	Percentage of Population Fully 💉
20–29	2689	795	77.2%	71.9%
30–39	3176	881	78.3%	77.4%
40–49	3303	635	83.9%	80.9%
50–59	2200	359	86.0%	84.4%
60–69	2200	187	92.2%	86.9%
70–79	1384	100	93.3%	92.8%
80–89	540	61	89.9%	91.2%
90+	142	20	87.7%	89.7%
TOTAL	TOTAL	TOTAL	AVERAGE	AVERAGE
20–90+	15634	3038	86.0%	84.4%

Source 1: https://data.gov.il/dataset/c -19/resource/9b623a64-f7df-4d0c-9f57-09bd99a88880
Source 2: https://datadashboard.health.gov.il/C -19/general

(b) According to an announcement by the German government, "The risk of an unwanted and serious side effect after the Corona-inoculation is very small. It resides in the vicinity of 0.02% and thereby affects one in every 5, 000 people on average."

> Nebenwirkungen treten in der Regel in kurzem Abstand zur Impfung auf.
>
> Das Risiko einer unerwünschten und schwerwiegenden Nebenwirkung nach einer Corona-Schutzimpfung ist sehr gering. Es liegt gerade einmal bei 0,02 Prozent und betrifft damit durchschnittlich eine von 5.000 Personen. Die bisherigen

How does this estimate, if it is at all honest, compare with the IFR in the age groups 0-19 and 20-29? [4 pts]