NAME:

Math 250 Exam 2

Instructions: WRITE YOUR NAME CLEARLY. Do as many problems as you can for a maximal score of 100. Note that you must do at least 10 problems correctly to get 100. Write neatly and legibly in the space provided. SHOW YOUR WORK!

Core Problems

1. a) An **epicycloid** is the path traced by a point on a circle of radius r that rolls on the *outside* of a fixed circle of radius R. By placing the moving circle initially so that it and the fixed circle are initially tangent at (R, 0) and by having it roll counterclockwise in the xy-plane, find a parameterization for this curve. [6 pts]

b) What is your parameterization in the special case when R = 6 and r = 2? [2 pts]

c) Modify your parameterization in (b) so that the epicycloid would lie on the plane (1, 2, 3) + $\frac{s}{\sqrt{18}}$(-1, 1, 4) + $\frac{t}{\sqrt{2}}$(1, 1, 0), with (1, 2, 3) as its center

[2 pts]

2. Let $f(t) = (t,\ -2t^2,\ t^3)$. Compute

a) $f^{'}(t)$ [4 pts]

b) $Df(t)$ [Hint: BE CAREFUL!] [6 pts]

3. A fashionable woman on high heels is running away from an angry bear in a hilly forest. She believes that the best way to get away from the bear is by climbing up as steeply as possible.

a) If the surface of the hill in meters is described by $f(x, y) = 10 - 3x^2 - y^2$ and she is currently in position (0, 1, 9), in what direction should she run? [4 pts]

b) Unfortunately, her expensive high-heel shoes will break if the slope is higher than 1. In what direction(s) should she move if fashion is as important to her as life itself? [Hint: There are 2 solutions] [6 pts]

4. Let $h(u,v) = (u+v\ ,\ u^2 - v^2)$, $g(s,t) = (\ln s\ ,\ s+t\ ,\ t^3)$, and $f(x,y,z) = (x^3 + y^2\ ,\ 3xz)$.

a) Compute the Jacobian J($f \circ g \circ h$)(1, 0) using the chain-rule. [6 pts]

b) Find D($f \circ g \circ h$)(1, 0) (u, v) [4 pts]

5. Let S = {(x, y, z) ; $x^4 + y^4 + z^4 = 1$} and $f(x,y,z) = xyz$.

a) Find $\left.\dfrac{\partial f}{\partial x}\right|_S$ [6 pts]

b) Compute $\left.\frac{\partial f}{\partial x}\right|_S (0, 0, 1)$ [4 pts]

6. Find all points on the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ that are closest to the origin (0, 0, 0). You may assume that $0 < a < b < c$. [10 pts]

7. Find the **third-order Taylor polynomial** for the function $f(x, y) = Sin(3x + 2y)$ about the point (0, 0). [10 pts]

8. The pharaoh Ramesses II has commissioned you to glaze his pyramid in gold. The pyramid stands 146 meters tall and has a square base with sides of length 230 meters. After glazing the pyramid uniformly, you find that the thickness of the pyramid at its base has increased by **2 millimeters** and that the pyramid's height has increased by **1 millimeter**. Approximately how much of the pharaoh's gold did you use? [10 pts]

9. Suppose $u(s,t) = tsf(s + e^t, s - e^{-2t})$ where $f(x, y)$ is some scalar-valued function. Compute $\frac{\partial u}{\partial t}$. [10 pts]

10. Find an infinite series expansion for $f(x, y) = Sin(x^2 + 2y)$.
[Hint: try u-substitution]. [10 pts]

Extra-Credit

11. The path of the space station orbiting the planet Solaris is given by $g(t) = (4 + Cos\left(\frac{\pi}{8}t\right), 8 - Sin\left(\frac{\pi}{8}t\right))$, where t is measured in years. For some time now, the crew members of the space station were behaving erratically until, a few weeks ago, status reports from the station have ceased entirely. It is suspected that they all went mad. As a trained stellar psychologist, you are sent to the space station to investigate what has happened. Because the trip is lengthy, you are to be frozen in a pod and loaded into a rocket. The rocket will carry the pod on a parabolic path $f(t) = (t, t^2)$ and then, at the appropriate time s, release the pod for docking with the space station automatically. When should you program the rocket to release the pod and when do you expect to arrive at the station? [10 pts]

12. Let $g : R \to R$ and $f : R^2 \to R$ be continuous. Define

$$F(x) = \int_0^{\int_0^x g(s)\,ds} f(x,t)\,dt$$

Find $\frac{dF}{dx}$ in terms of f and g. [Hint: Review the fundamental theorem of calculus and page 9 of my lecture notes on 2.6] [10 pts]

13. Earlier this semester we modified the definition of the derivative to encompass multivariate functions. Specifically, given a function
f : $U \subset R^n \rightarrow R^m$ and a point $a \in U$, we said that f is differentiable at a with derivative Df(a) if

$$\lim_{x \to a} \frac{\left\| f(x) - f(a) - Df(a)(x-a) \right\|}{\left\| x - a \right\|} = 0$$

for some linear map Df(a): $U \subset R^n \rightarrow R^m$.

Let $D^2 f(a)$ denote the 2nd derivative of f at a. What limit would you use to define the second derivative? What sort of "object" is it? [6 pts]

Justify the intuition behind your limit expression. [4 pts]