Solutions to H.W.#2

1. $a=(1,2,-1)$ $b=(3,6,-3)$

We know that $\|a\|\|b\|\cos\theta = a\cdot b$.

Hence $\theta = \cos^{-1}\left(\frac{a\cdot b}{\|a\|\|b\|}\right) = \cos^{-1}\frac{1\cdot 3+2\cdot 6+1\cdot 3}{\sqrt{1^2+2^2+1^2}\sqrt{3^2+6^2+3^2}} =$

$= \cos^{-1}\left(\frac{18}{\sqrt{6}\sqrt{3^2(1+2^2+1)}}\right) = \cos^{-1}\left(\frac{18}{6\cdot 3}\right) = \cos^{-1}(1) = 0.$

2. $a=(6,5,4)$ $b=(1,-1,1)$

$\theta = \cos^{-1}\left(\frac{a\cdot b}{\|a\|\|b\|}\right) = \cos^{-1}\left(\frac{6\cdot 1+5(-1)+4\cdot 1}{\sqrt{6^2+5^2+4^2}\sqrt{1^2+1^2+1^2}}\right) =$

$= \cos^{-1}\left(\frac{5}{\sqrt{77}\sqrt{3}}\right) \approx 1.24$ rad or about 70.8°.

3. $a=(8,5,1)$ $b=(-3,5,-1)$

$\cos^{-1}\left(\frac{0}{\|a\|\|b\|}\right) = \cos^{-1}(0) = \frac{\pi}{2}$

In other words $a \perp b$.

4. Recall that $p_{w}(v) = \frac{v\cdot w}{\|w\|^2}w$.

Hence $p_{(-1,-2,-1)}(4,6,-5) = \frac{(4,6,-5)\cdot(-1,-2,-1)}{(-1)^2+(-2)^2+(-1)^2}(-1,-2,-1) =$

$= \frac{-11}{6}(-1,-2,-1) = \frac{11}{6}(1,2,1)$

5. $P_{(3,9,27)}(4,8,16) = \dfrac{4\cdot 3+8\cdot 9+16\cdot 27}{3^2+9^2+27^2}(3,9,27)$

$= \dfrac{516}{819}(3,9,27)$

6. a) let $\vec{v} = (2,b,0)$ and $\vec{w} = (-3,2,1)$
then if we wish to have $\vec{v} \perp \vec{w}$ we must have
$\vec{v}\cdot\vec{w} = 0$ or $2\cdot(-3) + b\cdot 2 + 0\cdot 1 = 0$
in other words $-6+2b=0$ or $b=3$

b) if $\vec{v}$ is as before but $\vec{w} = (0,0,1)$ then
$\vec{v}\cdot\vec{w} = (2,b,0)\cdot(0,0,1) = 0$ hence any vector
of the form $2i+bj$ is orthogonal to k.

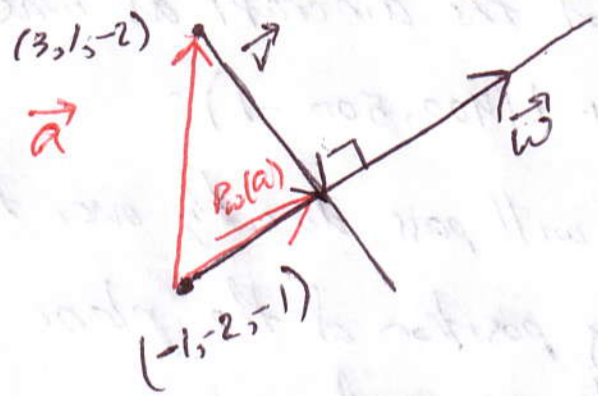
7. A vector of the form (a,b,c) is orthogonal to
$(1,1,1)$ if $a\cdot 1+b\cdot 1+c\cdot 1=0$ that is if $a+b+c=0$
Thus the set of all solutions to this problem is given by

$S = \{(x,y,z) : x+y+z=0\} = \{(x,y,z) : z = -(x+y)\}$

$= \{(x,y,-x-y) : x,y\in\mathbb{R}\} = \{(x,0,-x)+(0,y,-y) ; x,y\in\mathbb{R}\}$

$= \{s(1,0,-1)+t(0,1,-1) ; s,t\in\mathbb{R}\}$. That is the set of
all vectors orthogonal to $(1,1,1)$ is a plane spanned by
the vectors $(1,0,-1)$ & $(0,1,-1)$. These vectors are not parallel
(why?). Remark: there are infinitely many correct solutions.

8. We want to find the line $S(t)$ that goes through the point $(3,1,-2)$ and intersects the line $L(t) = (-1+t, -2+t, -1+t)$ $= (-1,-2,-1) + t(1,1,1) = (-1,-2,-1) + t\vec{w}$ at a right angle.
In particular $S(t) = (3,1,-2) + t\vec{v}$ where $\vec{v}$ may be obtained as follows:



Let $\vec{a}$ be the vector with tail at $(-1,-2,-1)$ and head at $(3,1,-2)$, then $\vec{a} = (3,1,-2) - (-1,-2,-1) = (4,3,-1)$.
We may think of $\vec{v}$ as a vector whose tail is at the head of $\vec{a}$ and whose head is at he head of

$P_w(a) = \dfrac{(4,3,-1)\cdot(1,1,1)}{1^2+1^2+1^2}(1,1,1) = \dfrac{6}{3}(1,1,1) = 2(1,1,1)$

$= (2,2,2)$

thus $\vec{v} = (2,2,2) - (4,3,-1) = (-2,-1,3)$

Observe that $\vec{v}\cdot\vec{w} = (-2,-1,3)\cdot(1,1,1) = -2-1+3 = 0$

so $\vec{v} \perp \vec{w}$ as desired.

Thus $S(t) = (3,1,-2) + t(-2,-1,3) =$

$= (3-2t, 1-t, -2+3t)$

Remark: the parametrization of the line is not unique. Consequently there are many correct solutions to this problem.

9. the position of the aircraft at time t is

$A(t) = (3,4,5) + t(400,500,-1)$

a) the pilot will pass directly over the airport when the $x\,y$ position of the plane will match the xy position of the airport.

that happens when $23 = x = 3 + t\cdot 400$ or when

$t = \frac{20}{400} = \frac{2}{40} = \frac{1}{20}$ (notice that $4 + \frac{1}{20}500 = 4 + 25 = 29$)

b) the height of the plane at time t is given by

$5-t$. when $t = \frac{1}{20}$ this becomes $\frac{5\cdot 20}{20} - \frac{1}{20} =$

$= \frac{99}{20}$ km above ground.