

HW. # 7

Homework problems are taken from textbook. The problems are color coded to indicate level of difficulty. The color **green** indicates an elementary problem, which you should be able to solve effortlessly. **Yellow** means that the problem is somewhat harder. **Red** indicates that the problem is hard. You should attempt the hard problems especially.

Evaluate the limit if it exists or state that the limit does not exist. In either case, justify your answer.

1. (a) $\lim_{(x,y) \rightarrow (1,-1)} \frac{x+4y}{1-x+y}$
- (b) $\lim_{(x,y) \rightarrow (0,\pi/2)} \frac{x+4y}{1-x+y}$
- (c) $\lim_{(x,y) \rightarrow (0,0)} \frac{1-e^{x^2+y^2}}{x^2+y^2}$
- (d) $\lim_{(x,y,z) \rightarrow (0,0,0)} \frac{x^3}{x^2+y^2+z^2}$
- (e) $\lim_{(x,y) \rightarrow (0,0)} \frac{y^2}{\sqrt{x^2+y^2}}$

Evaluate the limit if it exists or state that the limit does not exist. In either case, justify your answer.

2. (a) $\lim_{(x,y) \rightarrow (0,0)} \frac{xy}{\sqrt{x^2+y^2}}$
- (b) $\lim_{(x,y) \rightarrow (0,0)} \frac{xy}{x^2+y^2}$
- (c) $\lim_{(x,y) \rightarrow (0,0)} \frac{1-\cos(xy)}{xy}$
- (d) $\lim_{(x,y) \rightarrow (0,0)} \frac{xy^2-x^2y}{(x^2+y^2)^{3/2}}$
- (e) $\lim_{(x,y) \rightarrow (0,0)} \frac{x^3y}{x^6+y^2}$

Evaluate the limit if it exists or state that the limit does not exist. In either case, justify your answer.

3. (a) $\lim_{(x,y,z) \rightarrow (0,0,0)} \frac{x^3 + y^4 + z^5}{x^2 + y^2 + z^2}$
- (b) $\lim_{(x,y,z) \rightarrow (0,0,0)} \frac{xyz^2}{x^4 + y^4 + z^4}$
- (c) $\lim_{(x,y,z) \rightarrow (0,0,0)} \frac{x^2 + y^3 + z^4}{x^2 + y^2 + z^2}$
- (d) $\lim_{(x,y) \rightarrow (0,0)} \frac{x \cos(y) + y \sin(y)}{x^2 + y^2}$
- (e) $\lim_{(x,y) \rightarrow (0,0)} \frac{\sin(xy)}{x}$

Using the delta-epsilon definition of limit, prove that the following limits exist.

4. (a) $\lim_{(x,y) \rightarrow (1,5)} 2x - 5y$
- (b) $\lim_{(x,y,z) \rightarrow (3,-1,0)} 4x + 3y - z$
- (c) $\lim_{(x,y) \rightarrow (1,5)} (2x - 5y, 4x + y)$