HW. # 17

Homework problems are taken from several textbooks. The problems are color coded to indicate level of difficulty. The color green indicates an elementary problem, which you should be able to solve effortlessly. Yellow means that the problem is somewhat harder. Red indicates that the problem is hard. You should attempt the hard problems especially.

Evaluate the triple integrals by changing to cylindrical coordinates.

1. $\iiint_S 1+\frac{x}{\sqrt{x^2+y^2}}$; S is the solid bounded by the paraboloids $z=x^2+y^2$ and $z=1-x^2-y^2$.

2. $\iiint_S z$; S is the solid above the cone $z=\sqrt{x^2+y^2}$ and beneath the paraboloid $z=1-x^2-y^2$.

3. $\int_{-1}^{0}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}\int_{-1}^{3-x^2-y^2} z\,dz\,dy\,dx$

4. $\int_0^1\int_{-\sqrt{1/4-(y-1/2)^2}}^{\sqrt{1/4-(y-1/2)^2}}\int_{x^2+y^2}^{4}(x+y)\,dz\,dx\,dy$

5. $\int_0^{1/\sqrt{2}}\int_x^{\sqrt{1-x^2}}\int_{x^2+y^2-1}^{\sqrt{x^2+y^2}}(x^2+y^2)\,dz\,dy\,dx$

Evaluate the triple integral by changing to spherical coordinates.

6. $\iiint_S \sqrt{x^2+y^2+z^2}$; S is the solid in the first octant bounded by the coordinate planes and the sphere $x^2+y^2+z^2=9$.

7. $\iiint_S (x+y)$; S is the solid bounded above by the sphere $x^2+y^2+z^2=16$ and below by the cone $z=\sqrt{3x^2+3y^2}$.

8. $\iiint_S \frac{x}{\sqrt{x^2+y^2+z^2}}$; S is the solid in the first octant bounded by the planes $y = x$ and $y = \sqrt{3}x$, the cone $z = \sqrt{x^2+y^2}$, the plane z = 0, and the spheres $x^2+y^2+z^2 = 2$ and $x^2+y^2+z^2 = 8$.

9. $\int_{-2}^{2}\int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}}\int_{-\sqrt{4-y^2-z^2}}^{\sqrt{4-y^2-z^2}} (x^2+y^2+z^2)^{3/2}\,dxdzdy$

10. $\int_0^{1/\sqrt{2}}\int_y^{\sqrt{1/2-y^2}}\int_{\sqrt{x^2+y^2}}^{\sqrt{1-x^2-y^2}} \frac{1}{1+(x^2+y^2+z^2)^{3/2}}\,dzdxdy$

11. $\int_0^{3}\int_0^{\sqrt{9-x^2}}\int_0^{\sqrt{9-x^2-y^2}} xy\,dzdydx$

12. $\int_0^{1/\sqrt{2}}\int_y^{\sqrt{1-y^2}}\int_0^{\sqrt{1-x^2-y^2}} (z+y)\,dzdxdy$

Evaluate the triple integrals by making an appropriate change of coordinates.

13. $\iiint_S (x+z)^2$; S is the solid bounded by the planes $x+y+3z=0$, $x+y+3z=1$, $y+z=0$, $y+z=5$, $x+z=-1$, and $x+z=1$.

14. $\iiint_S \frac{yz+z^2}{x}$; S is the solid bounded by the planes z = x/3, z = 4x, y + z = 2, y + z = 5, x = 6, and x = 8.

15. $\iiint_S \cos y$; S is the solid bounded by the surfaces z = sin y +1, z = sin y – 1, and the planes y = 0, y = $\pi/2$, x + z = 0, and x + z = 1.