HW. # 15

Homework problems are taken from several textbooks. The problems are color coded to indicate level of difficulty. The color green indicates an elementary problem, which you should be able to solve effortlessly. Yellow means that the problem is somewhat harder. Red indicates that the problem is hard. You should attempt the hard problems especially.

Evaluate the triple iterated integral and describe the solid region over which you are integrating. In some instances, you may find it helpful to consult an integral table or use software that does symbolic integration.

1. $\int_0^1 \int_1^2 \int_2^3 \frac{ze^x}{y} dzdydx$

2. $\int_{-1}^1 \int_{-1}^1 \int_{x-1}^{y+1} xydzdxdy$

3. $\int_0^3 \int_0^{6-2y} \int_0^{(6-2y-x)/3} ydzdxdy$

4. $\int_0^1 \int_0^1 \int_0^{1+x^2} xyzdzdxdy$

5. $\int_0^2 \int_0^{\sqrt{4-x^2}} \int_0^{\sqrt{4-x^2-y^2}} xydzdydx$

Evaluate the triple integral of the given function over the indicated solid region. In some instances, you may find it helpful to consult an integral table or use software that does symbolic integration.

6. $f(x,y,z) = \sqrt{x+y+z}$; S is the solid bounded by the xy- and xz-planes, and the planes y = 2, x = 1, x = 4, and z = 5.

7. $f(x,y,z) = x$; S is the solid bounded by the paraboloids $z = x^2 + y^2$ and $z = 1-(x^2+y^2)$.

8. $f(x,y,z) = xy$; S is the solid bounded by the coordinate planes and the plane $x + 2y + 4z = 8$

9. $f(x,y,z)=x^2$; S is the solid bounded by the paraboloids $z=x^2+y^2$ and $z=8-x^2-y^2$. (Hint: An integration by parts may be necessary somewhere along the way.)

10. $f(x,y,z)=(2x-1)yz^2$; S is the solid in the first octant bounded by $x+y=1$ and $z^2=x^2+y^2$.

Find the volume of the indicated solid region by evaluating a triple integral.

11. The tetrahedron bounded by the coordinate planes and the plane x + y + z = 1

12. A sphere of radius r.

13. The solid inside the cylinders $x^2+y^2=a^2$ and $x^2+z^2=a^2$, where a is any positive number.

Find the mass of the solid with the given density.

14. S is the cube bounded by the planes x = ±1, y = ±1, z = ±1 and the density at a point is proportional to the distance from the point to the xy-plane.

15. S is the tetrahedron in exercise 11 and the density is xyz.

16. S is the solid bounded by the planes x = 1, x = 2, y = 1, y = 2, z = x + y and z = -2x – 3y; the density is proportional to the distance from the xy-plane.

In each of the triple integrals below, the solid of integration is x-, y-, and z-simple. Change the order of integration to represent the integral in terms of the other two types of solids.

17. $\int_{-1}^{0}\int_{0}^{1}\int_{1}^{2} f(x,y,z)\,dydzdx$

18. $\int_{0}^{2}\int_{0}^{4-2x}\int_{0}^{8-4x-2y} f(x,y,z)\,dzdydx$

19. $\int_{0}^{2}\int_{0}^{\sqrt{1-z^2/4}}\int_{0}^{3\sqrt{1-x^2-z^2/4}} f(x,y,z)\,dydxdz$

20. $\int_{0}^{2}\int_{x}^{2}\int_{0}^{5} f(x,y,z)\,dzdydx$