HW. # 12

Homework problems are taken from several textbooks. The problems are color coded to indicate level of difficulty. The color green indicates an elementary problem, which you should be able to solve effortlessly. Yellow means that the problem is somewhat harder. Red indicates that the problem is hard. You should attempt the hard problems especially.

Compute the Hessian matrix of the function at the point indicated.

1. $f(x, y) = 2x^2 - 3xy + 14y^2$; **a** = (1, 4)

2. $f(x, y) = \ln(x + y)$; **a** = (1, 1)

3. $f(x, y, z) = \tan^{-1}(x) + yz$; **a** = (0, 3, 1)

4. $f(x_1, x_2, x_3, x_4) = \dfrac{x_1 + x_2}{x_3 + x_4}$; **a** = (1, 1, 1, 1)

5. $f(x_1, x_2, ..., x_n) = x_1 + x_2 + ... + x_n$; **a** arbitrary

6. $f(x) = \|x\|^2$, $x \in R^n$; **a** is arbitrary

Find the second-degree Taylor polynomial for the given function at the point indicated.

7. $f(x, y) = 7x^2 + 4xy - 3y^2$; Point **a** =(1, -1)

8. $f(x, y) = x^3 + y^3$; Point **a** = (2, 3)

9. $f(x, y) = \sin(x^2 + y^2)$; Point **a** = (0, 0)

10. $f(x, y, z) = x + ye^z$; Point **a** = (1, 1, 0).

11. $f(x, y, z) = z \tan^{-1}\left(\dfrac{y}{x}\right)$; Point **a** = (1, 1, 1)

12. Show that if $f(x, y) = g(ax + by)$ and p is a Taylor polynomial of g centered at 0, then the corresponding Taylor polynomial of f centered at (0, 0) is given by $P(x, y) = p(ax + by)$

Use exercise 12 and your knowledge of Taylor polynomials for elementary functions to write down the Taylor polynomial of the indicated degree centered at (0, 0).

13. $f(x, y) = e^{x+y}$; m = 4, m = ∞

14. $f(x, y) = e^{(x+y)^2}$; m = 4, m = ∞

15. $f(x, y) = (x - y)e^{(x-y)/4}$; m = 4, m = ∞

16. $f(x, y) = \dfrac{1}{1 - x^2 - 2xy - y^2}$; m = 6, m = ∞