NAME:

Practice Math 250 Exam 3

Instructions: WRITE YOUR NAME CLEARLY. Do as many problems as you can for a maximal score of 100. Note that you must do at least 10 problems correctly to get 100. Write neatly and legibly in the space provided. SHOW YOUR WORK!

Core Problems

1. Evaluate $\iiint_B xyz$, where B is the unit cube (i.e. a cube of side-length 1 in the first octant, with one corner at the origin). [10 pts]

2. Evaluate $\int_0^{(\pi^{1/5})^2} \int_{\sqrt{x}}^{\pi^{1/5}} xSin(y^5)dydx$ [10 pts]

3. Let B be the solid region contained within the surface $x^2 + y^2 + z^2 = 4$ and lying above the surface $z = \sqrt{x^2 + y^2}$.

a) Identify or draw these surfaces. [3 pts]

b) Evaluate $\iiint\limits_B e^{-(x^2+y^2+z^2)^{3/2}}$ [7 pts]

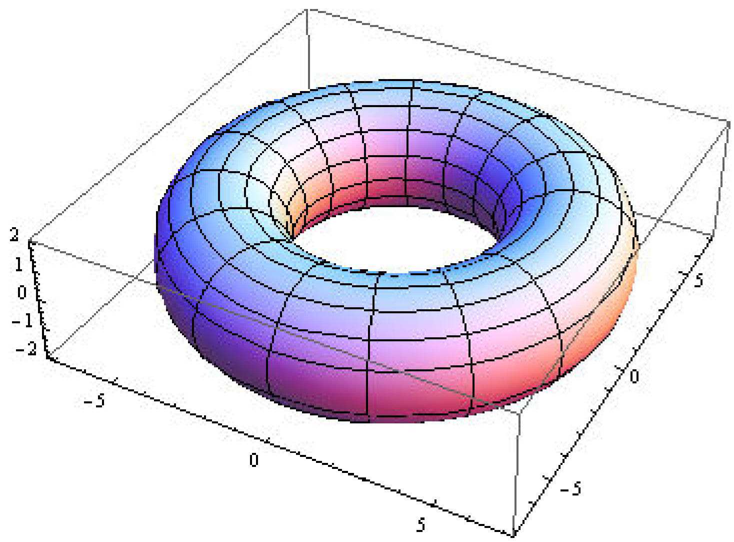
4. Let I = $\int_0^{\pi/2} \int_0^{2Cos(x)} f(x, y)dydx$.

a) Draw the region of integration and label it as x-simple, y-simple, both or neither. [4 pts]

b) Reverse the order of integration in I. [6 pts]

5. Let B be a region in the xy-plane bounded over the nonpositive x-axis (i.e. $x \le 0$) by the parabolas $y = x^2$, $y = x^2 - 4$ and the lines $3x - y = 0$ and $3x - y = 1$. Calculate $\iint_B \pi(2x-3)Cos(\pi x^2 - \pi y)$. [10 pts]

6. The doughnut-shaped solid S below is called a torus. It was generated by revolving a circular disc $(x-R)^2 + z^2 \le r^2$ in the xz-plane about the z-axis. Let $T : R^3 \rightarrow R^3$ be the linear map defined by T(x, y, z) = (x, y + z, 2y – z). If the volume of the torus is given by $V(S) = (\pi r^2)(2\pi R)$, what is the volume of the image of S under T, V(T(S))? [10 pts]



7. Evaluate $\iint_B \frac{-1}{\sqrt{1-x^2-y^2}}$ where B is the disc of ***diameter*** 1 centered at (0,½). [Hint: $\sqrt{x^2} = |x|$ <u>not</u> x] [10 pts]

8. Let B$\in R^3$ be a solid region in the ***first octant*** bounded by the elliptical cylinder $x^2 + y^2/9 = 1$ and the parabuloid $z = x^2 + y^2$. Set up $\iiint_B f$ as a z-simple iterated integral. [10 pts]

9. Evaluate the triple integral $\iiint_B \left(1 + \frac{x}{\sqrt{x^2 + y^2}}\right)$, where B is the solid bounded by the paraboloids $z = x^2 + y^2$, and $z = 8 - x^2 - y^2$ by changing to cylindrical coordinates. [10 pts]

10. Evaluate $\int_{-\infty}^{\infty} e^{-2x^2}\, dx$ [10 pts]

Extra-Credit

11. 3 numbers x, y, and z are chosen at random in the interval [0, 1]. What is the probability that the product of the first two numbers is bigger than the third?

[10 pts]

12. Evaluate $\iint_B 2x\ Sin^2\left(\ln\left(1+(xy)^2\right)\right)$, where B is the region in the xy-plane bounded by $x=|y|$, $x=-|y|$, y = 1 and y = -1 [Hint: Use symmetry]

[10 pts]