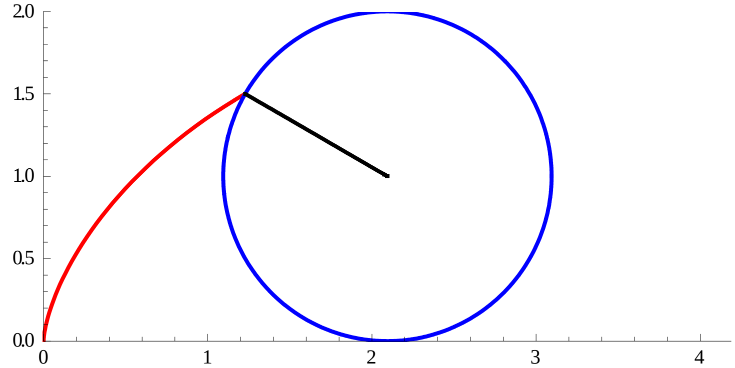
NAME:

Math 250 Practice Exam 2

Instructions: WRITE YOUR NAME CLEARLY. Do as many problems as you can for a maximal score of 100. SHOW YOUR WORK!

1. A **cycloid** is a curve that is traced by a point on a rolling circle that travels without slipping along the x-axis.



Find a parametric equation of the cycloid curve when the radius of the rolling circle is 2. [10 pts]

2. Let $p(t) = (t, \cos t, e^{2t})$.

(a) Compute $p'(0)$ [2 pts]

(b) Compute $Dp(0)$ [4 pts]

(c) If $p(t)$ represents the position of a particle at time t, what is the physical interpretation of your calculations in (a) and in (b)? [4 pts]

3. Let $k(x, y, z) = (x^2 - y^2 + z, 3x - y - z)$. Compute $Dk(1, 0, -1)$ [10 pts]

4. Let $f(x,y) = \int_{\pi}^{xy} \frac{\sin t}{t}\, dt$. Compute $Df(\pi/2, 1)$. [10 pts]

5. Let $f(x,y) = x^2 + \cos y$. Find the equation of the tangent plane to the graph $z = f(x,y)$ at the point $(1, \pi/2)$. [10 pts]

6. Let $g(x, y) = x^2 + xy$, $p(s, t) = (s^2 - t, s^2 + t)$, and set $u = g \circ p$. Find $\frac{\partial u}{\partial s}(1, 1)$. [10 pts]

7. Define $u = g \circ f$ where $f(x, y) = (3x + y, -2x + 3y, x + y)$ and $g(x, y, z) = x^2y + z$. Compute $Du(1, 0)$. [10 pts]

8. Let $f(x,y) = x^2y + \sin(\pi xy)$ and let $\vec{u}$ be a direction vector parallel to $\boldsymbol{i} + \boldsymbol{j}$. Calculate the directional derivative $D_{\vec{u}}f(1,-1)$ [10 pts]

9. A thrill-seeking family took their 87 year-old grandma on a hiking trip to the national paraboloid hill whose landscape is the graph $z = 5 - x^2 - y^2$ (in miles). If her current position is $(2,0,1)$, in what direction(s) should the poor old grandma head to avoid further climbing (i.e. avoid changing altitude)? [10 pts]

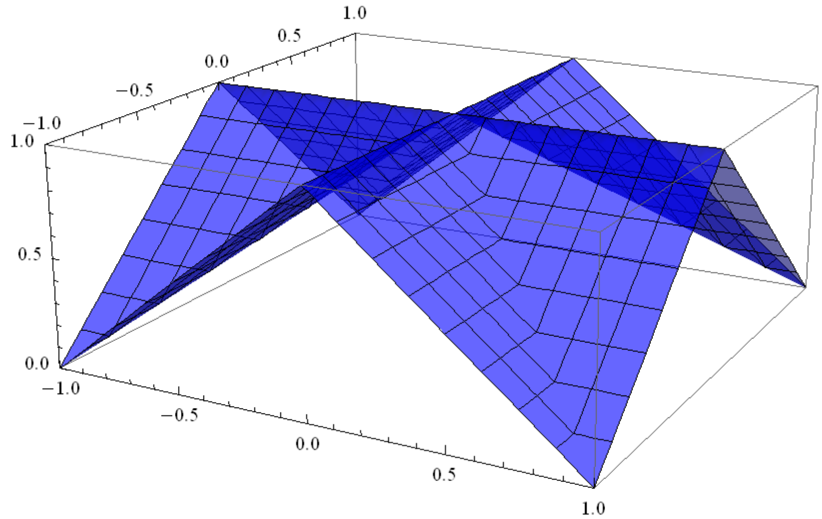
10. The point $(1, 0, -1)$ is a solution to the equation $x^2 z + ye^z = -1$.

(a) Does the equation define z as the implicit function $z(x, y)$ for x, y in the vicinity of $(1, 0)$? Explain. [4 pts]

(b) Compute $\frac{\partial z}{\partial x}$ at $(1, 0, -1)$ [6 pts]

Extra-Credit

11. The "Victorian cottage roof" is the graph of the function $f(x, y) = 1 - \min\{|x|, |y|\}$ is shown below:



(a) Using your geometric intuition or using the formula of f, compute $\frac{\partial f}{\partial x}(0,0)$ and $\frac{\partial f}{\partial y}(0,0)$. [2 pts]

(b) Using part (a) what would be your formula for $Df(0,0)$? [3 pts]

(c) According to your intuition, is f differentiable at (0,0)? Is the function obtained in part (b) the derivative of f at (0, 0)? [5 pts]

12. Use chain rule to derive the expression for quotient rule. In particular, if $f, g : \mathbb{R}^n \to \mathbb{R}$ are differentiable at $\vec{a} \in \mathbb{R}^n$ with $g(\vec{a}) \neq 0$, then $D(f/g)(\vec{a})(\vec{x}) = \frac{g(\vec{a})Df(\vec{a})(\vec{x}) - f(\vec{a})Dg(\vec{a})(\vec{x})}{[g(\vec{a})]^2}$. [10 pts]

13. Let $f: \mathbb{R}^2 \to \mathbb{R}$ be given by

$$f(x,y) = \begin{cases} \dfrac{x^2 y}{x^2 + y^2} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$$

(a) Is f continuous at $(0,0)$? [2 pts]

(b) Do all the directional derivatives $D_{\vec{u}}f(0,0)$ exist at $(0,0)$? [3 pts]

(c) Is f differentiable at $(0,0)$? [5 pts]

Justify all your assertions.