

NAME:

Math 250 Practice Exam 1

Instructions: WRITE YOUR NAME CLEARLY. Do as many problems as you can for a maximal score of 100. Note that you must do at least 10 problems correctly to get 100. Write neatly and legibly in the space provided. SHOW YOUR WORK!

Core Problems

1. a) Write a parametric equation of a line that goes through the point $(1, -5, 2)$ in the direction of the vector $-2\mathbf{i} + 3\mathbf{k}$. [2 pts]

b) Write a parametric equation of a line that goes through the points $(1, 2, 3)$ and $(-1, 0, 2)$. [2 pts]

c) Write a parametric equation of a line that goes through $(1, 2, 3)$ and is parallel to the line parameterized by $\mathbf{I}(t) = (1 + 2t, -t, 4 + 2t)$ [2 pts]

d) Are the lines parameterized by $\mathbf{I}(t) = (2 + 5t, -1 + t, 3 - 4t)$ and $\mathbf{S}(t) = (1 + 2t, -2t, 2 + 2t)$ perpendicular to each other? Why or why not? [2 pts]

e) Is the set $P = \{(1, 2, 3) + s(1, 7, -2) + t(-2, -14, 4); s, t \in (-\infty, \infty)\}$ a plane or a line? Why? [2 pts]

2. Sketch or describe the graph of $f(x, y) = \sqrt{x^2 + \frac{y^2}{9}}$ [10 pts]

3. Let $S = \{(x, y); x^2 + y^2 = 1\}$ and suppose $T(x, y) = (6x, 2y)$. What sort of curve is $T(S)$? What is the area enclosed by $T(S)$? [10 pts]

4. A triangle has vertices at $(1, 1)$, $(2, 3)$, and $(-1, 0)$. Find the area of this triangle. [10 pts]

5. Let $p = (1, 2, 3)$, $\mathbf{v} = 2\mathbf{i} - \mathbf{j} + 3\mathbf{k}$, and $\mathbf{w} = -3\mathbf{i} + 2\mathbf{j} - \mathbf{k}$.

a) Write a parametric equation of the plane spanned by the vectors \mathbf{v} and \mathbf{w} and going through point p . [5 pts]

b) Write an equation of this plane using only the coordinate variables x , y and z . [5 pts]

6. Let $T(x, y, z) = (3x + 2y - z, 6y, x - z)$ and $S(x, y, z) = (2x + y + z, z, 3z)$. Find:

a) $(T + S)(x, y, z)$ [1 pt]

b) $(S - 2T)(x, y, z)$ [1 pt]

c) $(TS)(x, y, z)$ [2 pts]

d) The matrix of T [2 pts]

e) The matrix of S [2 pts]

f) The transpose of the matrix of S [2 pts]

7. Let $A = \begin{pmatrix} 3 & 1 \\ 2 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 0 & 4 \\ 2 & 1 & -2 \end{pmatrix}$. Compute or state that the operation is not defined.

a) AB [5 pts]

b) BA [5 pts]

8. Find the inverse of the linear map $T(x, y) = (3x + y, 2x + y)$ [10 pts]

9. Does $\lim_{(x,y) \rightarrow (0,0)} \frac{2xy}{x^2 + y^2}$ exist? Justify your answer. [10 pts]

10. a) Suppose $\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = -2$ and $\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ b_{31} & b_{32} & b_{33} \end{vmatrix} = 1$. What is

$$\begin{vmatrix} 3a_{21} & 3a_{22} & 3a_{23} \\ a_{11} & a_{12} & a_{13} \\ b_{31} - a_{31} & b_{32} - a_{32} & b_{33} - a_{33} \end{vmatrix} ? \quad [6 \text{ pts}]$$

b) Let $\mathbf{v} = 6\mathbf{j} + 3\mathbf{k}$ and $\mathbf{w} = 2\mathbf{i} + 10\mathbf{j} + 8\mathbf{k}$. Find the projection of \mathbf{v} onto \mathbf{w} ,
 $P_{\mathbf{w}}(\mathbf{v})$ [4 pts]

Extra-Credit

11. Prove using the delta-epsilon definition of limit that
 $\lim_{(x,y,z) \rightarrow (1,-2,0)} 6x + y - 3z = 4$.

[10 pts]

12. Find the distance from the point $(1, 5, 6)$ to the line parameterized by
 $\mathbf{l}(t) = (1 + 2t, -1 + 10t, 3 + 8t)$

[10 pts]

13. Sketch or describe the graph of $f(x, y) = e^{-(y-2x)^2}$ [10 pts]