NAME:

Math 155 Practice Exam 1

Instructions: WRITE YOUR NAME CLEARLY. Do as many problems as you can for a maximal score of 100. SHOW YOUR WORK!

1.   Find $(f^{-1})'(2)$ given that $f(x) = x^3 + 3\sin x + 2\cos x$.   [10 pts]

2.   Differentiate $y = \dfrac{(x^3+2x^2)^4 \sec^2 x}{x^{1/3}}$ [Hint: Be smart about it!]   [10 pts]

3.   Differentiate $y = (\tan x)^{\sin x}$ [10 pts]

4. Evaluate the integral $\int \frac{2^x}{2^x+1} dx$ [10 pts]

5. The half-life of cesium-137 is 30 years. Suppose we have a 100-mg sample.
   (a) Find the mass that remains after t years.   [6 pts]

   (b) How much of the sample remains after 100 years?   [2 pts]

   (c) After how long will only 1 mg remain?   [2 pts]

6. Evaluate $\int_0^{\sqrt{3}/4} \frac{dx}{1+16x^2}$ [10 pts]

7. Compute $\lim_{x \to \infty} (e^x + x)^{1/x}$ [10 pts]

8.   Evaluate the integral $\int_0^1 \frac{r^3}{\sqrt{4+r^2}} \, dr$.   [10 pts]

9.   Evaluate the integral $\int_1^4 e^{\sqrt{x}} \, dx$.   [10 pts]

10. Find $\int \tan^6 x \sec^4 x \, dx$. [10 pts]

Extra-Credit

11.   Evaluate $\lim_{x \to 0}(\cos x - \sin 2x)^{1/x}$ [10 pts]

12.   Find the infinite polynomial expansion for $f(x) = \tan^{-1} x$.

[10 pts]

13.   Suppose that some function f has the following properties. $f(x + y) = f(x) + f(y) + 5x^2y + 5y^2x$ and $\lim_{x \to 0} \frac{f(x)}{x} = -3$. Find $f'(x)$.

[10 pts]