Department of Mathematics
Brooklyn College

Final Examination— Fall 2003
Mathematics 3.3

Show all work. No credit will be given unless work is shown. All answers should be given exactly; decimal approximations will not be accepted. Do all problems in Part I and any four problems in Part II.

PART I: *Answer ALL questions in this part. (40 points)*

(14 pts) 1. Find dy/dx for each of the following:

(a) $y = (4x-2)^3(x^2+x)^4$ (b) $y = e^{x^2-1} + 4\cos 2x$

(c) $y = \dfrac{\sin x}{\ln x}$ (d) $y = x^{\tan x}$

(14 pts) 2. Find each of the following:

(a) $\displaystyle\int \left(8x^3 - \frac{3}{x^2}\right) dx$ (b) $\displaystyle\int \frac{x-1}{x^2-2x}\, dx$

(c) $\displaystyle\int_1^{e^2} \frac{(\ln x)^3}{x}\, dx$ (d) $\displaystyle\int_0^{\pi/4} \sin^2(2x)\cos(2x)\, dx$

(12 pts) 3. Consider the function

$$f(x) = x^3 - 9x^2 + 15x - 6.$$

(a) Find the intervals on which f is (i) increasing, (ii) decreasing, (iii) concave up, and (iv) concave down.

(b) Find the x-coordinates of the points where f has a (i) local maximum, a (ii) local minimum, or a (iii) point of inflection.

(c) Sketch the graph of the function f and indicate the intervals and points identified in Parts (a) and (b) of the problem. The graph does not have to be accurate, but the features indicated in Parts (a) and (b) have to be shown correctly (i.e., for example, in the interval where the function is determined to be concave up, you in fact have to draw a curve that is concave up).

Please Turn Over

All computer processing for this document was done under Red Hat Linux. $\mathcal{A}\mathcal{M}\mathcal{S}$-TEX was used for typesetting. PICTEX was used for the diagram, together with the programming language Perl for generating the data points.

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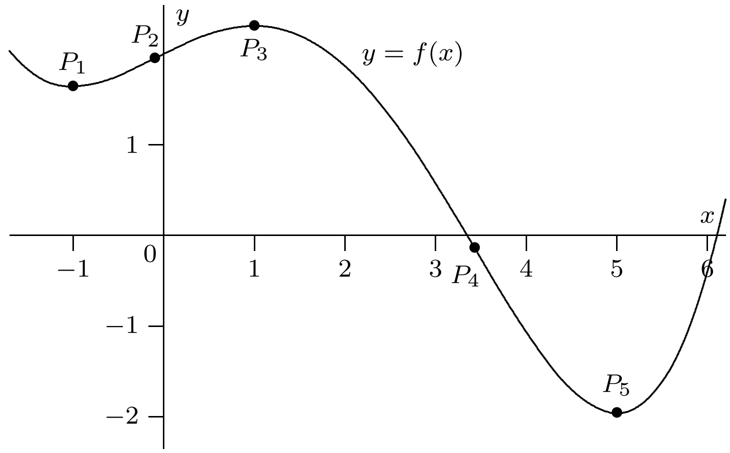
PART II: *Answer any FOUR of five questions in this part. (15 points each, a total of 60 points)*

4. (a) (i) Indicate whether each of the points P_1–P_5 in the enclosed graph identifies a local minimum, local maximum, or a point of inflection.



(ii) Sketch the derivative f' of the function f in the enclosed graph. The sketch need not be accurate, but it must correctly indicate where $f'(x)$ is positive, negative, zero, where it is increasing, and where it is decreasing.

(iii) Sketch the second derivative f'' of the function in the f in the graph. The sketch need not be accurate, but it must correctly indicate where $f''(x)$ is positive, negative, or zero.

(b) A rocket is launched vertically upward from a point 3 miles west of an observer on the ground. When the rocket is 4 miles above the ground, its distance from the observer increases at a rate of 0.4 miles per second. What is the speed of the rocket at this time?

5. (a) Let $f(x) = x + \dfrac{4}{x^2}$. Find the values of the (i) absolute maximum and the (ii) absolute minimum of $f(x)$ on the interval $[1, 4]$.

(b) Find the equation of the tangent line to the curve

$$x^2 + xy + y^2 = 7$$

at the point $(3, -2)$.

6. (a) *Using the definition* of derivative, find $f'(x)$ for $f(x) = \dfrac{x}{x+1}$.

   (b) A box with an open top must have a base twice as long as it is wide, and the total surface area of the box is to be 54ft^2. What is the maximum possible volume of such a box?

7. (a) Evaluate $\lim_{x \to +\infty} x(\sqrt{x^2+4} - x)$.

   (b) Find the area under the graph of $y = \dfrac{1}{x}$ between $x = e$ and $x = e^3$.

8. (a) A particle is moving along the x axis. Its acceleration at time t is $6t - \dfrac{4}{t^2}$. Its velocity at time $t = 1$ is 5, and its position at time $t = 1$ is $x = 2$. What is the position of the particle at time t?

   (b) Find $\lim_{x \to 9} \dfrac{x - 9}{\sqrt{x} - 3}$.

END OF EXAMINATION