Department of Mathmatics
Brooklyn College
Final Examination- Spring 2014
Math 1201: Calculus I
Total Points: 100

First Name:_____   **Last Name:**_____

Instructions: Solve <u>all three</u> problems in Part I and <u>any eight of the ten</u> problems in Part II. Show all work for full credit. Please <u>circle and label</u> your final answers. Scientific calculators permitted. No graphic calculators, cellphones, or other electronic devices permitted.

PART I: SOLVE <u>ALL THREE</u> PROBLEMS (44 pts). SHOW <u>ALL</u> YOUR WORK.

1. **[16 points]** Find $\frac{dy}{dx}$ for each of the following

   a) $y = (e^x + 4)\cos(2x)$ **b)** $y = \frac{\sqrt{x^2+1}}{(x+2)^5}$

   c) $y = (\tan(2x+1))^3$ **d)** $x^2 y^3 + \sec(x) = 8$

2. **[16 points]** Find each of the following .

   a) $\int \frac{2x^7+3}{x^5} dx$ **b)** $\int_1^e \left(\frac{1}{x} + 2^x\right) dx$

   c) $\int (2 + \cos(3x))^3 \sin(3x) dx$ **d)** $\int_1^4 \frac{e^{(4/x)}}{x^2} dx$

3. **[12 points]** Let $f(x) = \frac{x^4}{4} - 2x^2 + 2$

   (a)   i. Find the intervals where $f(x)$ is increasing, if any.
        ii. Find the intervals where $f(x)$ is decreasing, if any.

   (b) Find the local maximum and local minimum values of $f(x)$, if they exist (find both x and y values of each point).

   (c)   i. Find the intervals where $f(x)$ is concave up, if any.
        ii. Find the intervals where $f(x)$ is concave down, if any.
        iii. Find both the x and y values of any inflection points that exist.

   (d) Carefully sketch the graph of the function f and indicate the points identified in part (b) and (c) on the graph.

PART II: SOLVE ANY <u>EIGHT</u> OF THE FOLLOWING TEN PROBLEMS (7 pts each; Total 56 pts). SHOW <u>ALL</u> YOUR WORK.

4. Find the value of a that makes f continuous everywhere

$$f(x) = \begin{cases} \frac{x^2-4}{x-2}, & \text{if } x < 2; \\ ax^2 - x + 3, & \text{if } x \geq 2. \end{cases}$$

5. Use the *definition* of derivative to find $f'(x)$ for $f(x) = \sqrt{2 - 3x}$. (Obtaining $f'(x)$ by using the rules of differentiation will earn *absolutely no credit*.)

6. The following shows the graph of $f(x)$. Please copy the graph of $f(x)$ to your booklet and sketch the graph of $f'(x)$ on the same axes. Label $f'(x)$ where it is positive, negative or equal to 0.
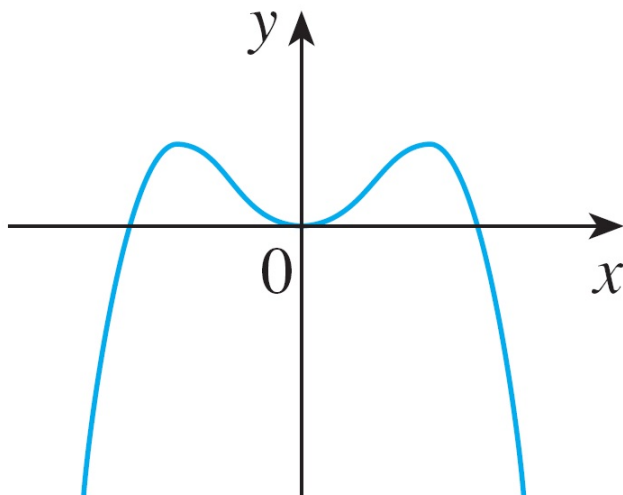
7. Find the equation of the tangent line to the graph of $y = (e^x - 3)^4$ at the point where $x = \ln 2$. Where does it intersect the x-axis (find both the x and y values for the intersection point)?

8. Suppose $f''(x) = e^x + 2 - \sin(x)$, $f'(0) = 3$, and $f(0) = 5$. Find $f(x)$.

9. Two cars start from point A at the same time. One travels west at 30 miles per hour and the other travels north at 40 miles per hour. How fast is the distance between them increasing 3 hours later?

10. A rocket is fired straight up from a tower of 240 feet above the ground with an initial velocity of 224 feet/sec. Assume that the downward acceleration due to gravity is -32 feet/sec^2. What is the velocity of the rocket when it hits the ground?

11. Find the area of the region that lies below the curve $y = \sin(2x)$ and above the x-axis between $x = \frac{\pi}{8}$ and $x = \frac{\pi}{6}$.

12. Find the absolute maximum and absolute minimum values of $f(x) = \frac{x}{x^2+1}$ on the interval $[0, 3]$.

13. A farmer has 240 feet of fencing and wants to fence off a rectangular field that borders a straight river. He needs no fence along the river. What are the width and length of the field that has the largest area? Justify your answers.

Kindly indicate on the cover of your examination the number of the problem in Part II that you omitted.

End of Examination