Math 150 Hand-In Assignment 4

The following questions are divided into two parts. All students should work on the suggested practice problems. The hand-in part is not mandatory. Its purpose is to identify and train the best and the most motivated students in the class and to help them achieve a deeper level of understanding of calculus. Some questions may be very hard and the student should not be discouraged. In mathematics one often battles with a problem for weeks without success, but this battle slowly makes one more durable and stronger mathematician.

Suggested Practice Problems

1. Differentiate the function using differentiation rules (derivative shortcuts)

   (a) $f(x) = 2^{40}$ [Hint: Be careful!]

   (b) $y = 3e^x + \dfrac{4}{\sqrt[3]{x}}$

   (c) $y = \dfrac{x^2 + 4x + 3}{\sqrt{x}}$

   (d) $F(y) = \left(\dfrac{1}{y^2} - \dfrac{3}{y^4}\right)(y + 5y^3)$

   (e) $f(x) = \dfrac{A}{B + Ce^x}$

   (f) $f(x) = \dfrac{1 - xe^x}{x + e^x}$

2. Find an equation of the tangent line to the given curve at the specified point.

   (a) $y = \dfrac{x^2 - 1}{x^2 + x + 1}$, (0, 0)

   (b) $y = \dfrac{e^x}{x}$, (1, e)

Problems to Hand-In

3. The limit $\lim_{h\to 0}\frac{\sqrt{1+h}-1}{h}$ corresponds to the derivative of some function $y = f(x)$ at some point x = a. What is the function f (x) and what is the point a?

4. Let f (x) be a function that is differentiable at x. In other words, $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = f'(x)$ exists.

   (a) What is $\lim_{h\to 0}\frac{f(x+h)-f(x)}{2h}$ in terms of $f'(x)$?

   (b) What is $\lim_{h\to 0}\frac{f(x-h)-f(x)}{h}$ in terms of $f'(x)$?

   (c) What is $\lim_{h\to 0}\frac{f(x+h^2)-f(x)}{h}$ in terms of $f'(x)$?

   (d) What is $\lim_{h\to 0}\frac{f(x+h)-f(x-h)}{2h}$ in terms of $f'(x)$?

5. Suppose that f (x) and g(x) are differentiable functions at the points x = a and x = b respectively. Furthermore, suppose that f (a) = g(b).

   (a) Express the limit $\lim_{h\to 0}\frac{f(a+h)-g(b+h)}{h}$ in terms of $f'(a)$ and $g'(b)$.

   (b) Use part (a) to compute the limit $\lim_{h\to 0}\frac{\sqrt{1+h}-e^h}{h}$.