

Math 150 Hand-In Assignment 3

The following questions are divided into two parts. All students should work on the suggested practice problems. The hand-in part is not mandatory. Its purpose is to identify and train the best and the most motivated students in the class and to help them achieve a deeper level of understanding of calculus. Some questions may be very hard and the student should not be discouraged. In mathematics one often battles with a problem for weeks without success, but this battle slowly makes one more durable and stronger mathematician.

Suggested Practice Problems

1. Find the limit or show that it doesn't exist.

(a) $\lim_{x \rightarrow \infty} \frac{x^2}{\sqrt{x^4 + 1}}$

(b) $\lim_{x \rightarrow -\infty} \frac{\sqrt{9x^6 - x}}{x^3 + 1}$

(c) $\lim_{x \rightarrow -\infty} (x + \sqrt{x^2 + 2x})$

(d) $\lim_{x \rightarrow \infty} (\sqrt{x^2 + ax} - \sqrt{x^2 + bx})$

(e) $\lim_{x \rightarrow -\infty} \frac{(3x^3 - 7x^2 + 2)^9 (-x^2 - x)^7}{(3x^2 + 12)^7 (5x - 1)^{27}}$

(f) $\lim_{x \rightarrow \infty} \frac{\sin^2 x}{x^2 + 1}$

Problems to Hand-In

2. Give an example where $\lim_{x \rightarrow 5} f(x)$ and $\lim_{x \rightarrow 5} g(x)$ do not exist, but $\lim_{x \rightarrow 5} (f(x) + g(x)) = 7$. [Hint: we did something similar in class to warn against applying limit laws incorrectly]
3. Give an example where $\lim_{x \rightarrow 5} f(x)$ and $\lim_{x \rightarrow 5} g(x)$ do not exist, but $\lim_{x \rightarrow 5} [f(x) \cdot g(x)] = 7$.

4. Recall that two functions f and g are the same function, if they have the same domain D and for every input $x \in D$, $f(x) = g(x)$. For example, the functions $f(x) = x^3 \cos^2(x) + x^3 \sin^2(x)$ and $g(x) = x^3$ are actually the same function because $\cos^2(x) + \sin^2(x) = 1$. In particular, f is a polynomial function, even though it is given by a formula that is not readily recognized as polynomial form.

(a) Explain why $f(x) = \sqrt{x}$ is not a polynomial.

(b) How do you know that $f(x) = \sqrt{2x^2 + 1}$ is not a polynomial? In other words, how do you know that there is no function $g(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_0$ such that $f(x) = g(x)$?