

Math 150 Hand-In Assignment 2

The following questions are divided into two parts. All students should work on the suggested practice problems. The hand-in part is not mandatory. Its purpose is to identify and train the best and the most motivated students in the class and to help them achieve a deeper level of understanding of calculus. Some questions may be very hard and the student should not be discouraged. In mathematics one often battles with a problem for weeks without success, but this battle slowly makes one more durable and stronger mathematician.

Suggested Practice Problems

1. (a) For what value of the constant c is the function f continuous on $(-\infty, \infty)$?

$$f(x) = \begin{cases} cx^2 + 2x & \text{if } x < 2 \\ x^3 - cx & \text{if } x \geq 2 \end{cases}$$

- (b) Find the values of a and b that make f continuous everywhere

$$f(x) = \begin{cases} \frac{x^2 - 4}{x - 2} & \text{if } x < 2 \\ ax^2 - bx + 3 & \text{if } 2 \leq x < 3 \\ 2x - a + b & \text{if } x \geq 3 \end{cases}$$

- (c) Use the Intermediate Value Theorem to show that the equation $x^4 + x - 3 = 0$ has a root in the interval $(1, 2)$.
- (d) Use the Intermediate Value Theorem to show that the equation $\sin x = x^2 - x$ has a root in the interval $(1, 2)$.
- (e) Suppose f is continuous on $[1, 5]$ and the only solutions of the equation $f(x) = 6$ are $x = 1$ and $x = 4$. If $f(2) = 8$, explain why $f(3) > 6$.
- (f) A Tibetan monk leaves the monastery at 7:00 AM and takes his usual path to the top of the mountain, arriving at 7:00 PM. The following morning, he starts at 7:00 AM at the top and takes the same path back, arriving at the monastery at 7:00 PM. Use the Intermediate Value Theorem to show that there is a point on the path that the monk will cross at exactly the same time of day on both days.

Problems to Hand-In

2. Use a $\delta - \varepsilon$ argument to prove that $\lim_{x \rightarrow 5} (-2x + 1) = -9$

3. Use a $\delta - \varepsilon$ argument to prove that $\lim_{x \rightarrow -2} (2x^2 - 5x + 1) = 19$