

NAME:

Fall 2020 Math 1201 Exam 1

Instructions: WRITE YOUR NAME CLEARLY. Do as many problems as you can for a maximal score of 100. SHOW YOUR WORK!

1. Suppose $\lim_{h \rightarrow 0} \frac{f(1+5h) - f(1)}{h} = 10$.

Find $\lim_{h \rightarrow 0} \frac{f(1+h) - f(1)}{h}$

[10 pts]

2. Use the squeeze theorem to evaluate $\lim_{x \rightarrow 0} f(x)$ where

$$f(x) = \begin{cases} \frac{e^x - e^{-x}}{x} & \text{if } x \text{ is irrational} \\ \frac{\sin 2x}{x} & \text{if } x \text{ is rational, } x \neq 0 \end{cases}$$

[10 pts]

3. Evaluate $\lim_{x \rightarrow -\infty} x e^{\frac{1}{x}} - x$ [10 pts]

4. Find an equation of the tangent line to the curve $\frac{10x \cos(x) e^x}{(1 + \sec^2(5x))} + 11$ at the point $(0, 11)$. [Hint: Shortcuts aren't always short] [10 pts]

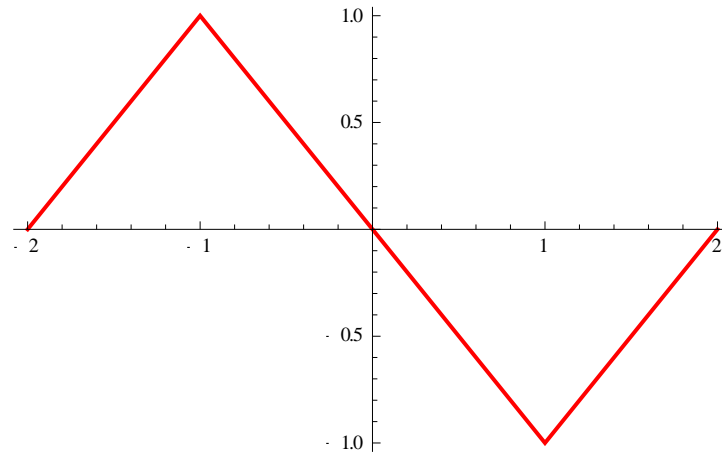
5. Find the derivative of the function $f(x) = \sqrt{1 - 7x}$ using the definition of the derivative at any given point x . [10 pts]

6. If $\lim_{x \rightarrow 5} f(x) = 27$, find $\lim_{x \rightarrow 5} \frac{\sqrt[3]{f(x)} - 3}{f(x) - 27}$ [10 pts]

7. Use the intermediate value theorem to show that the square root of any positive number is defined. [10 pts]

8. Find a value k that will make $f(x) = \begin{cases} -x^2 & \text{if } x \leq 2 \\ k^2 - 2kx & \text{if } x \geq 2 \end{cases}$ a continuous function. [10 pts]

9. The graph of the function $y = f(x)$ is displayed below



Draw the graph of $y = f'(x)$.

[10 pts]

10. Evaluate $\lim_{\theta \rightarrow 0} \frac{\sin(5\theta) + \tan(3\theta)}{\sin(\theta) + \tan(15\theta)}$ [10 pts]

Extra-Credit

11. Prove by means of a delta-epsilon argument that if $\lim_{x \rightarrow 2} f(x) = 9$ then $\lim_{x \rightarrow 2} \sqrt{f(x)} = 3$ [10 pts]

12. You plan to invest 1 (thousand) dollars in one of the following accounts:

Account 1: Money stays dormant for one year and doubles at the end of the period.

Account 2: Money grows by a factor of $1/2$ every $1/2$ year

Account 3: Money grows by a factor of $1/3$ every $1/3$ year

Account 4: Money grows by a factor of $1/4$ every $1/4$ year

Account C: Money grows continuously as the limit of the process described above.

What is the best option? How much will money should you have on your account if you choose option C?

[10 pts]