

Lecture 0: Why Do We Need To Study And Understand Proofs?

At the beginning, middle, and end of a typical series of calculus lessons, the most frequent questions are

1. Is it on the exam?
2. How is it useful in real life?

More frequently than not, the first question betrays fear, and the second question exposes your indifference to the subject with the accompanying desire to have it declared unimportant. Mathematics is a lot more poetry than bookkeeping. Indeed, the poem below adequately summarizes what you will experience in this class:

Alexander Blok Demon

Behind me you must go, behind me,
My slave obedient and true;
The sparkling mountain-ridges find me
In flight unfaltering with you.

Above abysses I shall take you,
Bottomless pits of mystery;
And there, while futile terrors shake
you,
Is inspiration's strength for me.

Amid the ether's flaming shower
I do not let you swoon, but show
My shadowy wings and sinewy power
To lift you and not let you go.

Upon the hills in white resplendence,
Upon the unstained meadow-ground,
In beautiful divine attendance
My fire shall strangely burn around.

Know you how frail is that delusion
By which mankind is tricked, how
small
Is the poor pitiful confusion
That by wild passion's name we call?

When shadows gather in the even
And my enchantment senses you,
You wish to fly aloft to heaven
Through fiery deserts of the blue.

I gather you in my embraces
And raise you up with me afar
To where a star is like earth's places
And earth's not different from a star.

Then stricken dumb with admiration,
New universes you can see,
Sights unbelievable, creation
Made by my playful fantasy.

In fear and strengthlessness you shiver;
I hear you whisper: «Let me go!»
You from my soft wings I deliver
And smile upon you, «Fly below!»

Beneath my smile divinely winning,
In an annihilating flight,

Like a cold stone, you flutter, spinning
Into the glittering void of night.

<https://www.youtube.com/watch?v=Yt6EinZBoks>

There are a few things to add in reference to the second question. What role does higher math play in your everyday life? Has it ever played one? Were you aware of this role? You were asked to watch the movie "12 Angry Men". How certain are you of your common sense? Do you believe you would make a good jury? Can you think without bias? Is there anything in this world that you perceive objectively?

Let's start with a simple common sense question:

Question: You are invited to a household known to have 2 children. You have never seen these children and have no prior knowledge about them. As you park your car, you notice that one child is playing in the yard and that she is a girl.

a) What is the probability both children are girls?

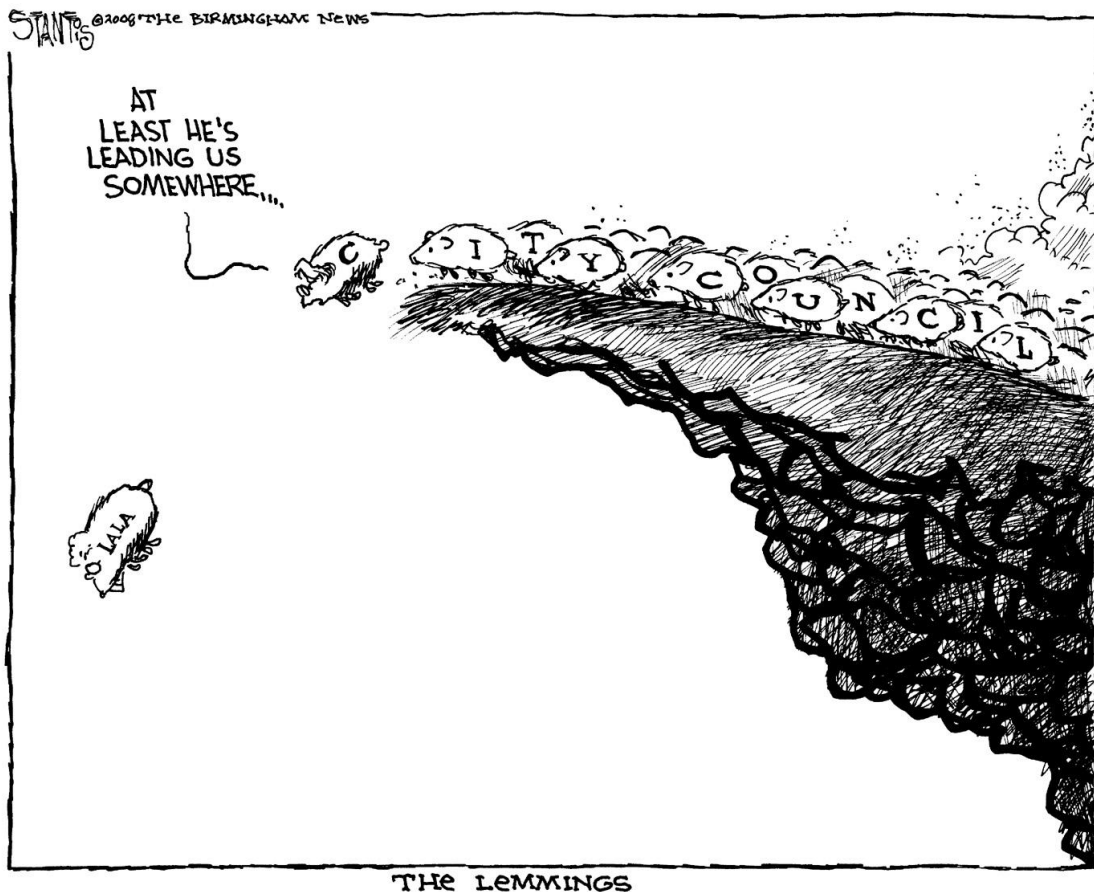
Answer: Since you don't know if you are observing the youngest or oldest child, there are 3 equally likely possibilities (g, g), (b, g), (g, b), where the first coordinate stands for oldest child and the next coordinate represents the youngest child. Thus the desired probability is $1/3$.

- b) Her mother comes to greet you and as she yaps away you suddenly hear "...Oh this child was born on Wednesday...". What is the probability that both children are girls?

Answer: It is a mistake to think that the gender of the children is the only relevant information. If we assume that each child is equally and independently likely to be born a boy or a girl and is equally and independently likely to come into existence on any given day of the week, then the relevant objects are vectors

of the form (gender of oldest, the day of its birth, gender of youngest, the day of its birth) = (x, t, y, s) . The given information tells us that we either have the case (g, w, y, s) or (x, t, g, w) . Each of the two vectors represents a total of $2 \times 7 = 14$ possibilities with only one overlap, namely (g, w, g, w) . Thus we are faced with a total of $2 \times 14 - 1 = 27$ possibilities, of which vectors of the form (g, w, g, s) and (g, t, g, w) represent outcomes in which both children are female. The event (g, w, g, w) is the only one in common. Thus, there are $2 \times 7 - 1 = 13$ possibilities. Hence the desired probability is $13/27$

This might seem like a toy problem, one that cannot possibly matter in real life. Doesn't it? Well, I think the lesson to take home here is this: It is very easy to get into situations where your thinking is utterly incorrect. And if you forego thinking altogether, you follow those who lead you like lemmings off a cliff.



Let's make the conversation more dangerous. Why are we speaking through ZOOM rather than in person?

Believe it or not, the answer has very much to do with calculus. We have closed schools, movie theaters, restaurants and other "nonessential" businesses. Many have postponed surgeries and diagnostic medical procedures for things that can wait (like cancer), and the responsible and thoughtful citizens among us commute everywhere with a face mask on. Why?

To the best of my knowledge, the logistic model was used to simulate the spread of the novel Corona virus. If we rescale so that 1 represents the maximal possible population that would be affected by the virus without preventative treatments

(such as vaccines), then the logistic model arises as a solution to the differential equation

$$\frac{dP}{dt} = rP(1 - P)$$

Where P stands for the size of infected population at time t and r represents the virulence rate.

During the better part of the class we will study the derivative and its properties. For now, please take it on belief that the solution to this differential equation is obtained through the following computation:

$$\int \frac{dP}{P(1 - P)} = \int r dt$$

By the method of partial fractions this simplifies to

$$\int \left(\frac{1}{P} + \frac{1}{1 - P} \right) dP = \ln \left(\frac{P}{1 - P} \right) = rt + c$$

Upon solving for P and simplifying, we obtain

$$P(t) = \frac{P_0 e^{rt}}{1 - P_0 + P_0 e^{rt}}$$

where P_0 represents the "patient 0 population" at the beginning of the infection.

Obviously I don't anticipate that you understand anything at all from this derivation. I just want you to note that solving this rather simple equation (differential equations are generally very scary monsters) involves

- Properties of differentiation (The tangent line problem)
- Integrals (The area problem)
- natural logarithms and this very special number e . Consider this: You have ignored this number all your life, but in a very definite sense, it determines on which side of the bed you will wake (if at all). That it will determine the grade that you'll get, isn't even worth mentioning.

Question: What do you think this equation says about your likelihood of getting COVID? If you are susceptible to getting sick, do you think that wearing masks, socially distancing, and using all sorts of other precautions will help you dodge this virus?

Answer: According to this model, $P(t)$, the fraction of the susceptible population will satisfy

$$\lim_{t \rightarrow \infty} P(t) = \lim_{t \rightarrow \infty} \frac{P_0 e^{rt}}{1 - P_0 + P_0 e^{rt}} = 1$$

Limits will be the first topic that we will tackle. By the time this semester is over, you should be able to carry out such calculations with ease.

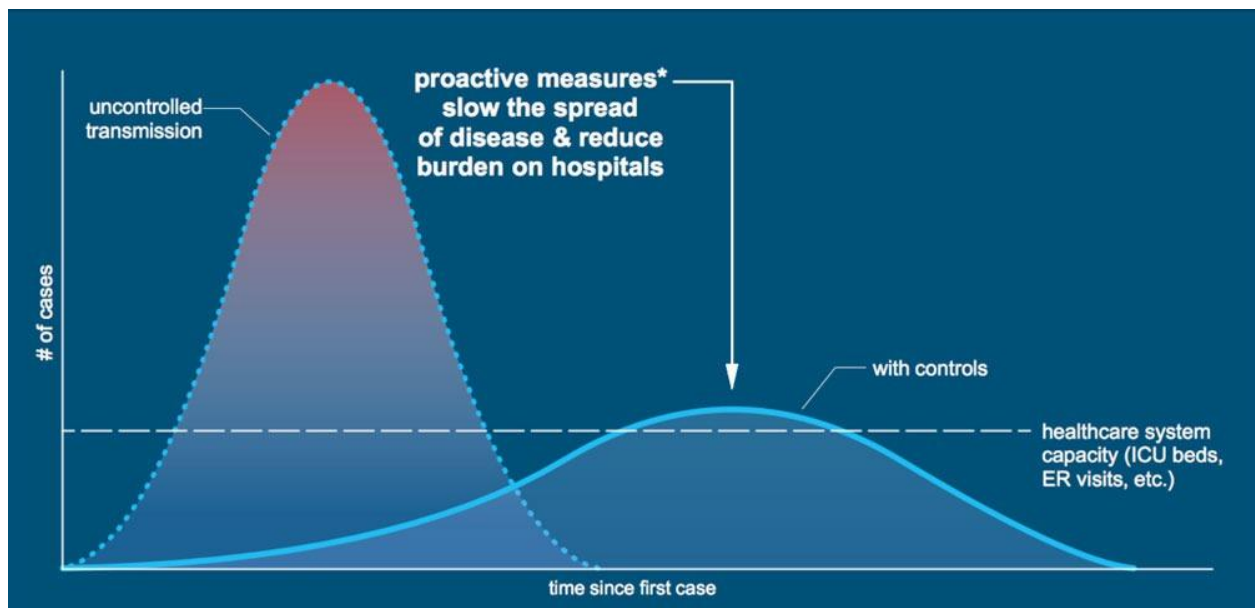
This limit means that as time goes on and on, eventually everyone who could potentially get ill will get infected. Heuristically speaking, I know someone who avoided going out of the house entirely for the better parts of 3 months and still caught the virus.

Question: So if everyone who is "destined" to get COVID will eventually get it, what is (or was supposed to be) the purpose of the lockdown?

Answer: To the bright and progressive people who believe in science, the answer is sweet and simple.



And to those who get a minor fit of apoplexy at the utterance of this oxymoron "I believe in science", the following diagram explains the rationale:



At the start of the crisis, it has been argued that

- The virulence rate of this illness is alarmingly higher than that of the common influenza virus.
- COVID is 5 to 10 times more lethal to the host.
- Huge inflow of critically ill patients will overwhelm the healthcare system.

The mathematical model (the logistic function) used to forecast how the infection will spread across its target population predicts

- Everyone within the target population will eventually become infected. In other words, the total number of people that will get sick will be the same no matter what.
- Preventative measures, like perhaps, masks, social distancing, etc might reduce the virulence rate r so that the total number of sick patients will spread over broader intervals of time.
- The flat and the sharp curves both have the same areas = same number of sick patients. However, the flat curve insures that not all the patients are in the hospital at once.
- # Flatten The Curve = Don't come to the hospital critically ill all at once.

Simply put, I have often imagined that the lockdown was declared by one of my pre-med students. Flatten the curve is a single-variable optimization problem and only a few of the medical track students go on to Calculus III.

It is popularly believed that life cannot go back to normal until a vaccine against COVID is developed. What you might not be aware of is that several prominent virologists, among which is Prof. Dr. Sucharit Bhakdi, consider this effort to be the intellectual equivalent of trying to shove an elephant through the eye of the needle. Discussing the peculiarities of the immune system is beyond our scope, but if you are curious, you can see Prof. Bhakdi's explanation in this excerpt, which I translated

<https://www.youtube.com/watch?v=xDYUPjqNerU&t=2s>

Just a few days ago, this was released. Very interesting! Watch carefully and watch to the end:

<https://www.youtube.com/watch?v=4uo2KGiSjrW>

Let us return to mathematics and ask a few silly questions:

Question: If I test positive for Corona Virus, how likely is it that I carry the infection? To make the question precise, suppose that a COVID test is 99% reliable. That is, if the patient has COVID, the test comes positive 99% of the time, and if the patient isn't afflicted with the illness, a false positive is 1% likely. Let's say that the reliability is beyond dispute. If a patient tests positive, what is the likelihood that the patient has the virus?

Answer: It is impossible to ascertain from the information given. To answer this question, we need an accurate estimate of the prevalence of the disease in the general population. In other words, we need to know the probability p that a randomly selected individual carries the illness. For instance, if $p = 0$, then no matter what this test says, I can be certain that I am not infected.

This p is generally unknown. Instead of ramping up tests you have to gather a random sample of sufficient size. From this sample, a beta distribution will be generated. From this beta distribution the value of p can then be estimated. Understanding beta distributions involves lots and lots of calculus!

https://en.wikipedia.org/wiki/Beta_distribution

Have you heard the expression, a philosopher is a blind man in a dark room with no windows, searching for a black cat that is not there?

All tests are fallible. If you rump up testing blindly, you will find something, even if it is not there!

Question: Is COVID much more infectious than influenza? Why isn't there a lockdown every winter?

Answer: Forecasts for the spread of COVID infections were predicting exponential growth (Makes me wonder why they are not already predicting hurricanes months before they hit the shore!).

<https://www.youtube.com/watch?v=fgBla7RepXU&t=33s>

<https://www.youtube.com/watch?v=Kas0tlxDvrg&t=346s>

You might also find these videos about mask wearing quite interesting :

<https://www.youtube.com/watch?v=0Tp0zB904Mc>

<https://www.youtube.com/watch?v=npXP5wqNzaI&t=309s>

I would be very interested to know what you think.

We will study exponential functions later in the course. There are many fables that try to illustrate the enormous rapidity of exponential growth. In one such fable, a king reluctantly agreed to repay a wise man for his service and asked him to name his price. The wise man replied: I am a modest man. Take a grain of gold and place it on one square of the chess board, take two grains and place them on

the next square, place 4 grains on the 3rd square, 8 grains on the 4th square, and so on. The miserly king was very happy with this arrangement, not realizing that

$$1 + 2 + 4 + 8 + \dots + 2^{64} = 2^{65} - 1$$

or 368,934,881,474,191,032,31. Since these grains are visible, this wise man is asking for about as much gold as would be necessary to create a full size gold replica of the entire planet.

The exponential function of base 2 looks like this: 2^x .

Exponential growth means growth that corresponds to increased exponent values for some a^x for $a > 1$.

Let's see if you're already good at recognizing such growth. On the Israeli news, it was reported that COVID positive numbers were rising significantly after the ease of their lockdown¹. Let's say the number of newly identified COVID infections over 7 days has been : 2, 4, 8, 16, 32, 64, 128.

True or False? Based on this evidence, we can conclude that the infection is spreading exponentially. Why?

¹ In Israel very stern measures were taken during the initial few months of the outbreak. One couldn't even leave the house to walk farther than 200 meters. This plunged the economy to its lowest point in 40 years, but what's economy when lives are at stake!? At any rate, one can hold his breath only for so long. The lockdown was eased. The alarmed media started reporting increases in positive tests and heralding the coming of the second wave. If you are interested, you can see one report that I translated <https://www.youtube.com/watch?v=odN4HsIBBko>

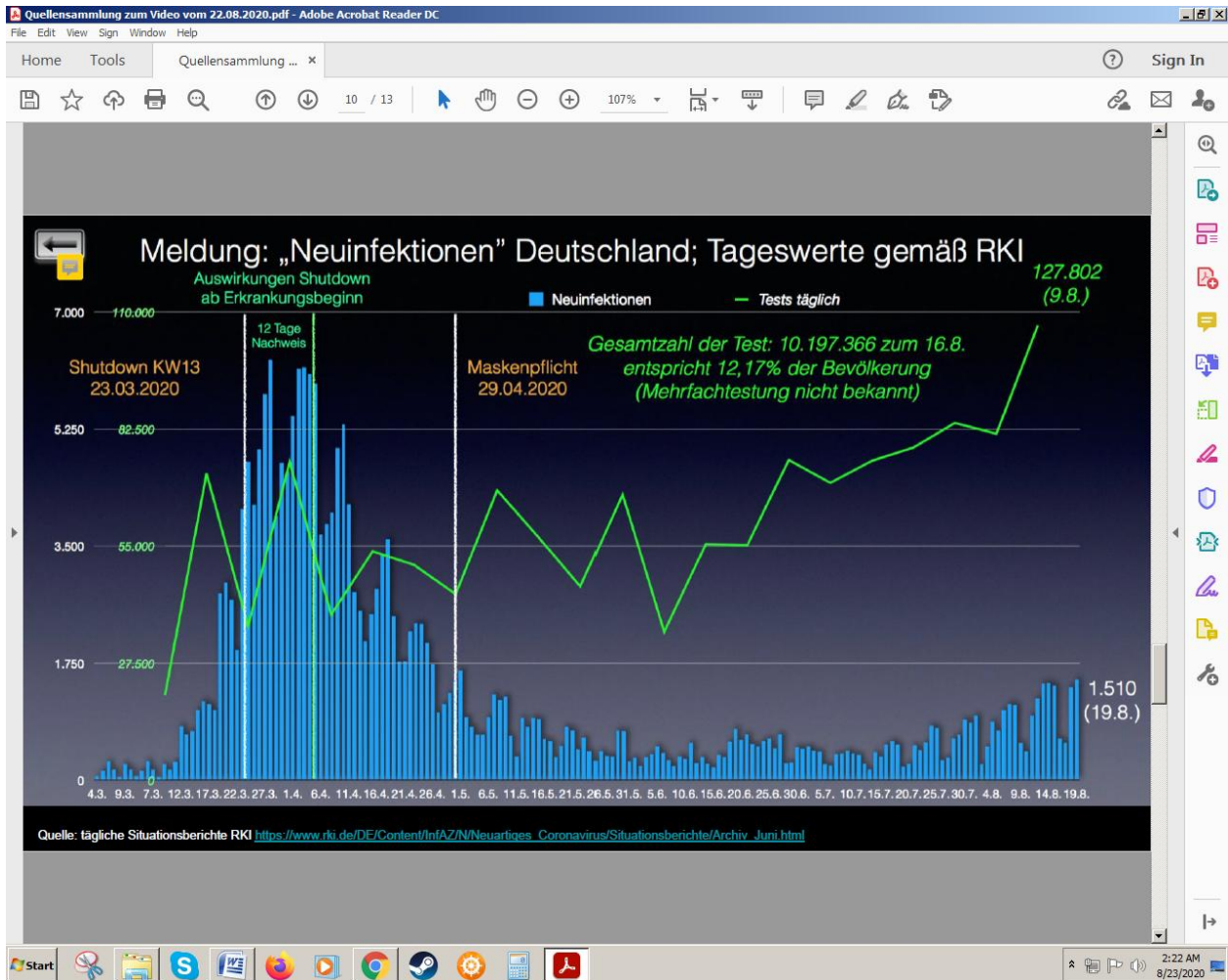
False! Imagine that on day one we carried out 10 tests of which 2 were positive. We get alarmed and on the second day, we carry 20 tests and find 4 positive. My God!! On the 3rd day, we carry 40 tests and find 8 positive. Keeping this up, we carry out 640 tests on day 7 and find a whopping 128 possibly infected individuals.

However,

$$\frac{2}{10} = \frac{4}{20} = \frac{8}{40} = \dots = \frac{128}{640}$$

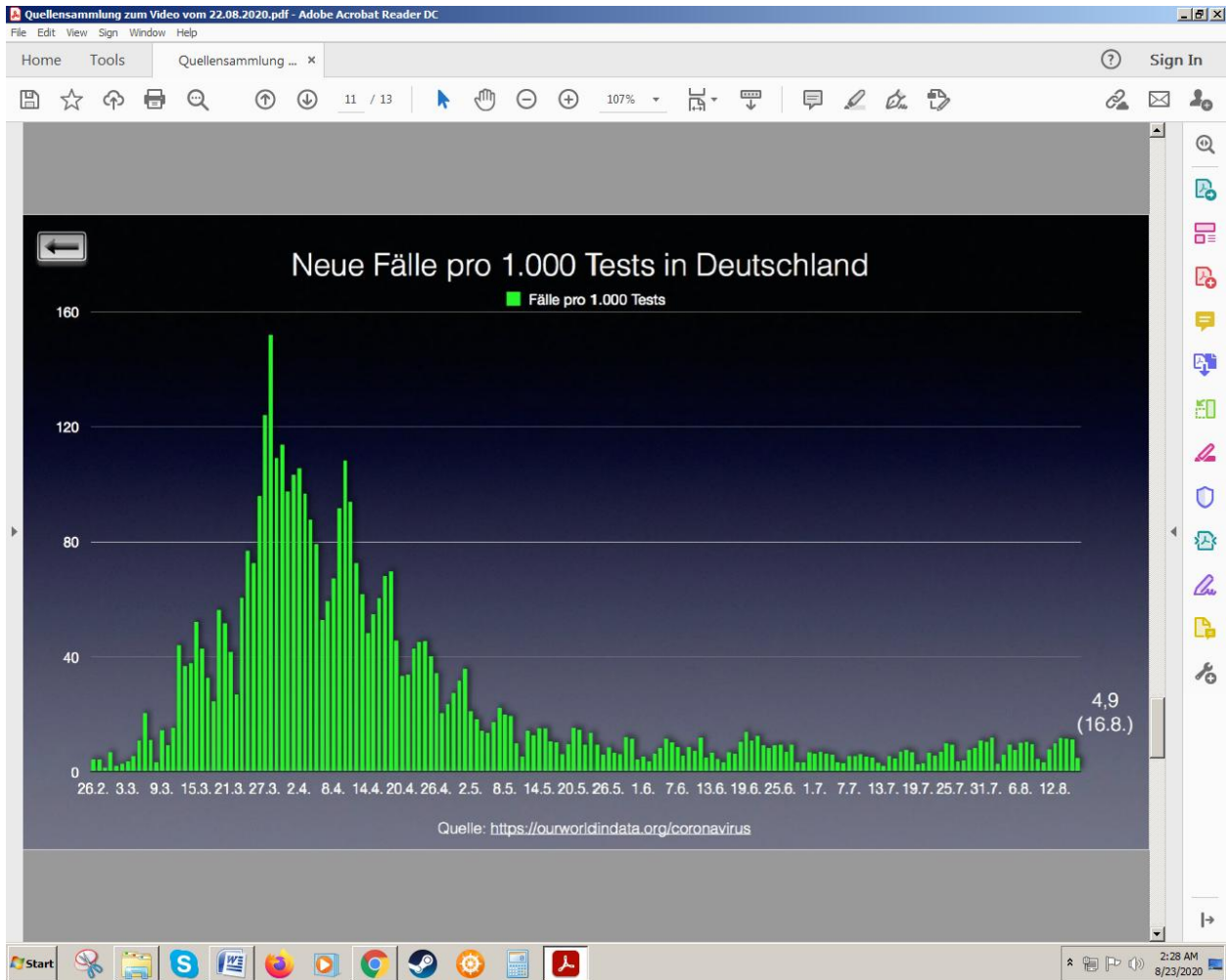
The size of the affected population is stable! Without normalizing the data, which in this case means dividing the number of positive cases by the total number of tests, the extrapolation is utterly meaningless.

Interestingly enough, this little step is frequently omitted. After all, fractions, math, and proofs are boring, right? Germany, Bavaria especially, has experienced particularly stringent lockdowns, just as Israel did. New laws, empowering the government to take drastic action in times of pandemic, are in the process of being drafted. Major protests took place on August 1 and one will take place on August 29. One reason for this is the creeping indication (or fear) that the German government will declare that the second wave has begun. You can see it in the diagram represented by the blue bars.



See how the blue bars are rising on the right? The blue bars represent the total positive tests for a given period. Now the green jagged line represents the total number of tests. Do you see how it rises on the right as well?

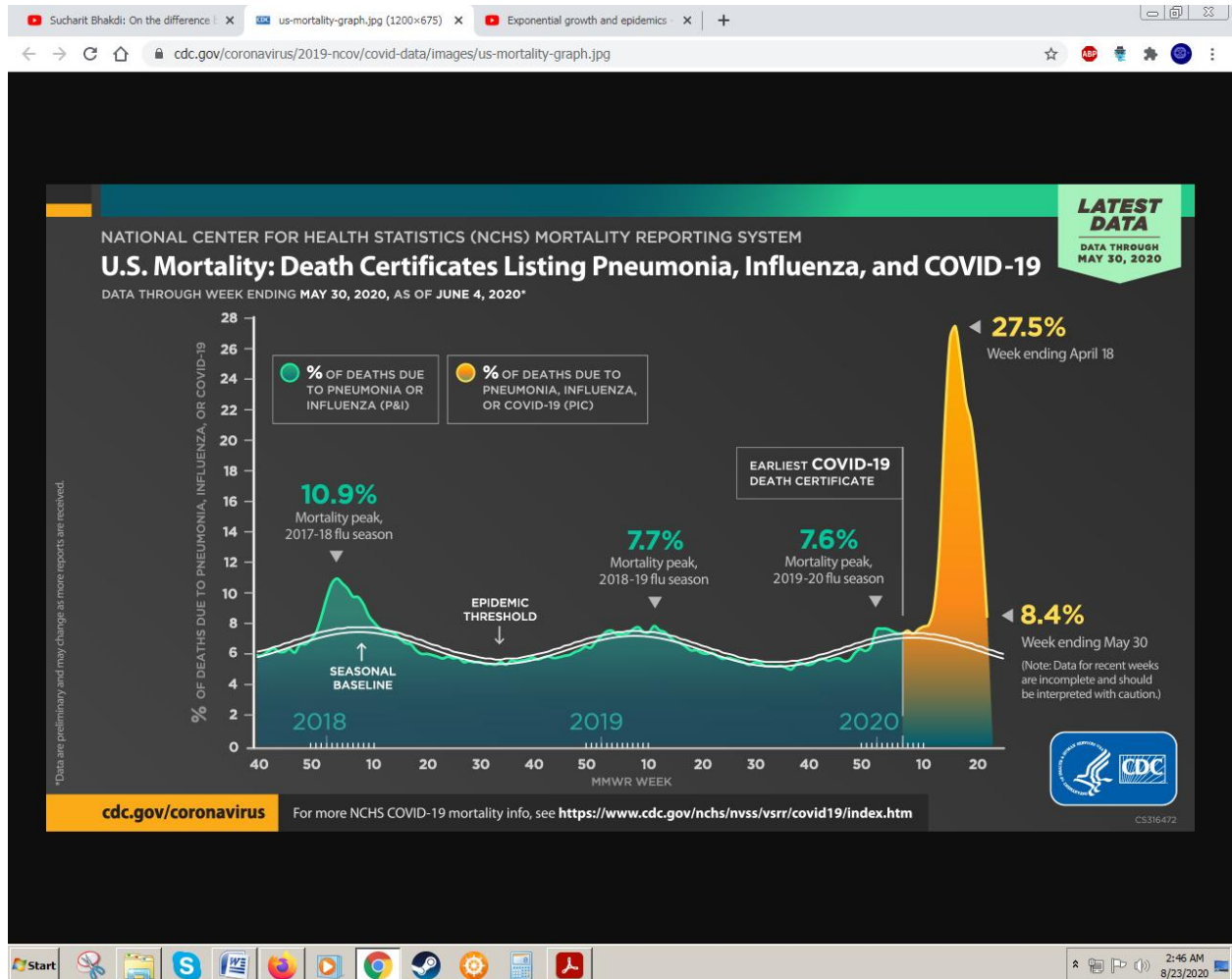
Now here is a picture of the normalized graph



This graph represents new positive COVID results per 1,000 tests. Coupled with the fact that these may be false positives, the tail on the right might be nothing but noise.

Question: Which is more lethal, influenza or COVID? How do you know?

Answer: According to the CDC, anywhere between 20,000 to 60,000 individuals die yearly from influenza. So far, the reported death toll from COVID-19 has been around 175,000. The CDC has also generated the following graph:



This graph shows a spike in the proportion of death certificates listing Pneumonia, Influenza, or COVID-19.

Now you are the jury. You are the 12 angry men. Armed with this evidence, do we convict the Corona virus of murder?

Here again, the devil is in the details. One is tempted to imagine succumbing to a viral illness as a straightforwardly understood process: One is healthy as a cucumber, one gets very ill, hospitalization is required, the patient's condition becomes critical, the last resort treatments (even the ventilator) are of no avail, the patient is declared dead and the virus that got him to the hospital is justly listed as the cause of his demise.

The truth is a little bit more complicated. One usually dies from multiple comorbid conditions. It is generally a difficult matter to determine someone's cause of death. With COVID-19, there has been strong international pressure to list the virus as the primary cause of death for any deceased person, on whose body the Corona virus was found or even suspected. In other words, if someone died in a motorcycle accident and had COVID, he could be listed as a COVID victim.

The scary CDC chart lists cause of death by proportion (compare with the charts in the previous question!). This mode of presentation is mathematically unsound. Consider the following scenario:

In a certain hospital, exactly 100 patients die in April. Their death certificates list various causes. Suddenly strict guidelines are issued to list the hospital bed as the primary cause of death whenever the patient expires on or in the vicinity of the mattress. How do you think the yellow spike of deadly hospital beds will look like? These graphs are meaningless without knowledge of total mortality rates (from all causes) over many years!

Why is this data not immediately mentioned in conjunction with COVID mortality numbers?

Following a helpful suggestion, I looked up the overall mortality statistics of great Britain. I was able to obtain records for the years 2009-2020.



About twice as many individuals died in April than the average mortality rate for the periods that I examined. This death toll was about 5 standard deviations from the norm, making this fluctuation statistically significant. In the above graph, you can see the big spike in the death rate.

The gentlemen that brought these data to my attention is of the opinion that the British government waited for the death rates to spike on purpose, because British citizens wouldn't otherwise believe in the efficacy of the lockdown. Does this information convince you that we are witnessing the epidemic of the century?

Prof. Bahkdi has some interesting things to say about the lethality of COVID in his latest video. It is interesting to note that the reported death toll in different

countries appears highly inconsistent. There were over 170, 000 reported deaths in the US compared to under 10, 000 in Germany. Is the difference to be attributed to effective lockdown measures in one country and poor decision making in the other?

There are unfortunately few countries without lockdown to carry out a comparison study. To my knowledge, Belarus has taken no measures whatsoever, but, being a dictatorship, their data is unreliable. Sweden has not imposed a lockdown. The death toll in Sweden is about 6,000 in a population of over 10 million people. By comparison, the Spanish influenza has taken over 34, 000 Swedish lives when their population was about 4 million.

The following videos might shed some light on what happened in NY.

Dr. John Ioannidis of Stanford University discusses his perspective.

<https://www.youtube.com/watch?v=cwPqmLoZA4s>

For a more visceral eyewitness account, you might find this video quite informative:

<https://www.youtube.com/watch?v=UIDsKdeFOmQ&t=409s>

My grandfather worked as a head of the biomedical department at Elmhurst Hospital and my brother did some volunteer work under him some years ago. These accounts suggest that certain hospitals were a major source for the spread of the illness and that faulty treatment and nosocomial infections within these hospitals were perhaps responsible for the amplified lethality.

I don't know about you, but this "wear a mask" business reminds me of my mother. I imagine saying to her: Mom, I am going to a duel. Where is my pistol? She: It's cold outside. Don't forget your sweater.